Complex Systems Methods — 12. Computation and Complexity at the edge of chaos

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Overview

1. Computation at the edge of chaos
   - Computation in physical systems
   - Langton's study of 1-d Cellular automata
2. Criticism
3. Further developments
4. Resumé

Computation at the edge of chaos — the original claim

1990 appeared in Physica D a paper called "*Computation at the edge of chaos: Phase transitions and emergent computation*" by Christopher G. Langton, where he asked the following question:

Under what conditions will physical systems support the basic operations of information transmission, storage, and modification constituting the capacity to support computation?

And his answer was:

Systems near a continuous ("second order") **phase transitions**, i.e. near **criticality**, between an **ordered** ("solid") and **chaotic** ("liquid") state are especially capable of computations.

Langton's study of 1-d CA: Computation

Two views on computation in CA's:

1. The CA rule is the program and the initial conditions are the data. Running the CA means to perform a computation.
2. Langtons view: The CA rule represent (some part of)the physical world. The initial configuration itself constitutes the computer, the program, and the data. Langton translates his original question into a new one, i.e. when it is possible to adopt this view to understand the dynamics of the CA.

In order to show that a CA is capable of universal computation one has to construct a Turing machine (**in** the CA), what corresponds to the second point of view.

Computation in physical systems

Langton identifies three crucial properties which are required in this case:

Memory: The physics must support storage of information. The dynamics must be able to preserve local information for an infinite time (at least in principle).

Transmission: The dynamics must provide the propagation of information in the form of signals.

Information Processing: Stored and transmitted information must be able to interact with each other

Langton argues that memory implies a diverging correlation time and transmision a diverging correlation length $\Rightarrow$ Criticality.

Wolframs universailty classes of CA dynamics

A continuous dynamical system interpretation of Wolframs universality classes:

Class	CA	Dynamical System
I	Homogeneous state	Fixed point
II	Periodic structures	Limit cycle
III	"Chaotic" aperiodic patterns	strange attractors
IV	Complex patterns and localized structures	Very long transients

Wolframs conjecture: Class IV cellular automata are computational universal, i.e. one can use them to contstruct a universal Turing machine.

Parametrizeing the CA space - the λ parameter

In order to study some kind of transition in the space of CA rules, one needs a paramtrization of this space. Langton proposed the so called "λ-parameter":

- There is a "quiescent state" s with the condition that an environment consisting only of cells in state s is mapped to s. This corresponds to the interpertation of all other states as excited or active states and the CA as a model of an excitable medium.
- If the single cell in the CA has K states and an environment containing N cells there are K^N different environments.
- There are n different environments mapped to the quiescent state s.
- The λ-parameter is defined as the fraction of environments, that are not mapped to the quiescent state:

$$\lambda = \frac{K^N - n}{K^N}$$

Qualitative Dynamics

Langton considered 1-d CA's with $K = 4$ local states and nearest and next neares neighbour interaction $N = 5$.

The rule space was searched by two different methods:

1. Random table method: λ is interpreted as the bias in the random selection of the rule table entry
2. Table walk through: Starting from $\lambda = 0$ entries in the rule table are replaced randomly.

Qualitative Dynamics

Langton considered 1-d CA's with $K = 4$ local states and nearest and next neares neighbour interaction $N = 5$.

Starting from a random initial condition:

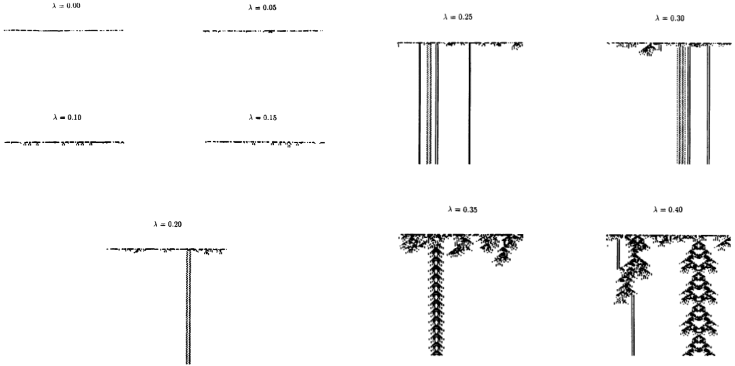

Fig. 1. Evolutions of one-dimensional, $K = 4$, $N = 5$ CAs from *fully* random initial configurations over $0.0 < \lambda \leq 0.75$. As λ is increased the structures become more complicated, and the transients grow in length until they become arbitrarily long at $\lambda \approx 0.50$. For $0.50 < \lambda \leq 0.75$, the transient lengths *decrease* with increasing λ, as indicated by the arrows to the right of the evolutions.

Qualitative Dynamics

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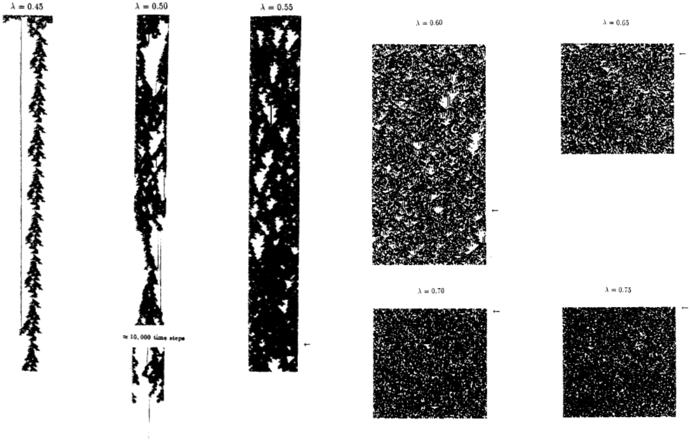


Langton considered 1-d CA's with $K = 4$ local states and nearest and next neares neighbour interaction $N = 5$.

Starting from a localized random state:

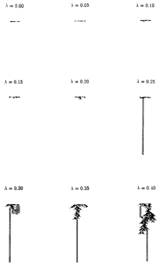
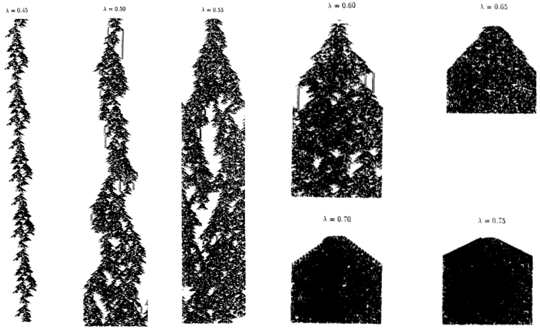


Fig. 2. Evolutions of one-dimensional, $K = 4$, $N = 5$ CAs from *partially* random initial configurations over $0.0 < \lambda \leq 0.75$. This series illustrates the change in the rate of spread of the dynamics from negative for $\lambda < 0.45$, to positive for $\lambda > 0.45$. For $\lambda \approx 0.45$, the dynamics is balanced between collapse and expansion, giving rise to particle-like solitary waves.



Quantitative characterization

Transient length:

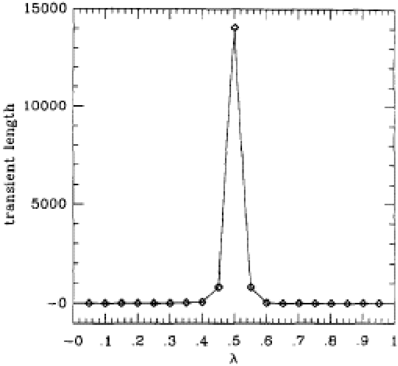


Fig. 3. Average transient length as a function of λ in an array of 128 cells.

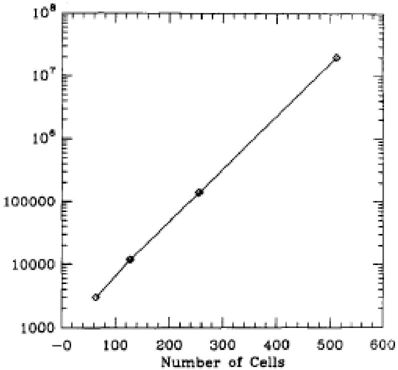


Fig. 4. Growth of average transients as a function of array size for λ = 0.50.

Single site entropy $H(x_i)$ and mutual information $MI(x_i(t+1) : x_i(t))$:

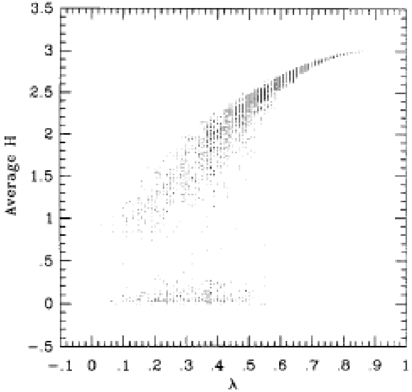


Fig. 6. Average single cell entropy $\overline{H}$ over λ space for approximately 10 000 CA runs. Each point represents a different transition function.

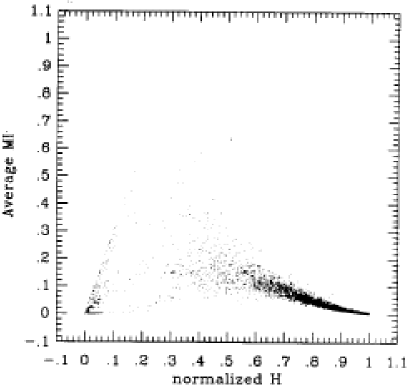


Fig. 14. Average mutual information versus average single cell entropy $\overline{H}$. The mutual information in this case is computed between a cell and itself at the next time step. The entropy is normalized to 1.0.

Quantitative characterization

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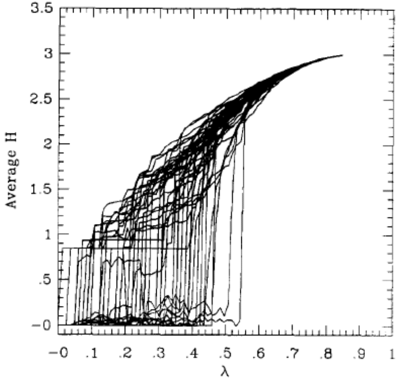
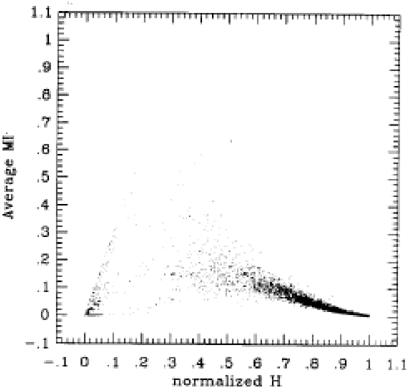


Fig. 8. Superposition of 50 transition events, showing the internal structure of fig. 6.



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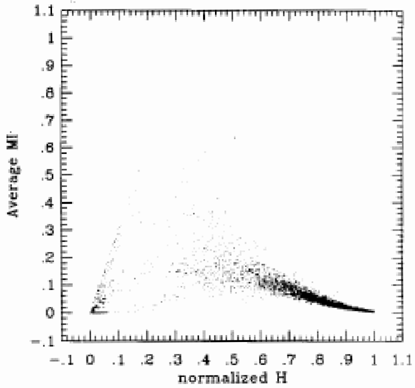
Quantitative characterization

Complexity and the "edge of chaos":



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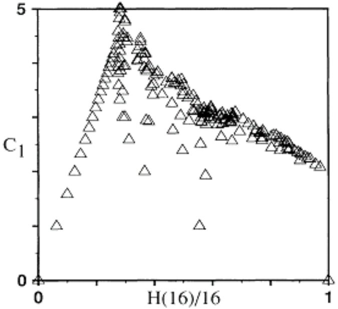
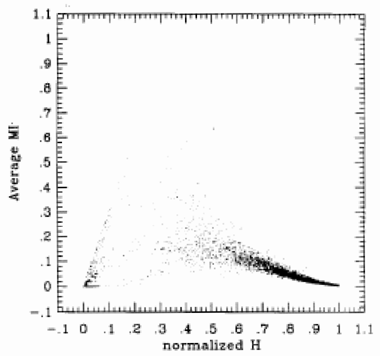


FIG. 2. Graph complexity C_1 vs specific entropy $H_1(16)/16$, using the binary, generating partition $\{[0,0.5),[0.5,1]\}$, for the logistic map at 193 parameter values $r \in [3,4]$ associated with various period-doubling cascades. For most, the underlying tree was constructed from 32-cylinders and machines from 16-cylinders. From high-entropy data sets smaller cylinders were used as determined by storage. Note the phase transition (divergence) at $H^* \approx 0.28$. Below H^* behavior is periodic and $C_a = H_a = \log(\text{period})$. Above H^*, the data are chaotic. The lower bound $C_a = \log(B)$ is attained at $B \rightarrow B/2$ band mergings.

Complexity and the “edge of chaos”:



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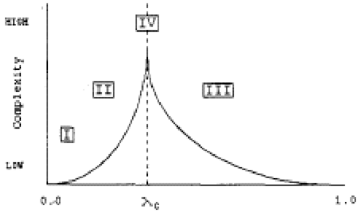


Fig. 16. Location of the Wolfram classes in λ space.

Computation at the "edge of chaos"

Langton's experiment on CAs:

- Ordered dynamics: no information transmission, low number of different states $\Rightarrow$ low memory capacity
- Chaotic dynamics: no memory
- At the edge of chaos: criticality, long range and long term correlations
  - High number of different states $\Rightarrow$ high memory capacity
  - Long transients: information storage, transmission and processing

Related ideas:

- "Life" at the edge of chaos: the game of life is supposed to have a critical value of λ
- Stuart Kauffman: Evolution drives living systems to the "edge of chaos" because there the systems are especially adaptive
- Per Bak: A large class of systems drives itself into a critical state — self-organized criticality (SOC)

Criticism

How well defined is the "edge of chaos?

- The use of the notion of "phase transition" by Langton was only metaphorically
- Qualitative changes as in the dynamics of dynamical system as a function of a control parameter are usually called a *bifurcation*, but not any bifurcation is a phase transition.
- No quantitative diagnosis for criticality.
- The parameter space of cellular automata is not continuous.
- There might be different dynamical regimes for one CA rule.
- Even in Langtons analysis no sharp transition was observed with most of the measures.
- There might be better measures for the analysis of the CA behaviour, e.g. excess entropy vs. entropy rate

- Packard (1988) in paper called "Adaptation toward the edge of chaos" evolved CA's using genetic algorithm to estimate whether there are more 0s or 1s in the initial condition and claimed that the CA's performing well in this computational task exhibit critical values of λ.
- Crutchfield, Mitchell et al. (1993) repeated this experiment and could not reproduce the result: instead they found that the rules evolved towards rules with λ values in the chaotic region and supported this finding with theoretical arguments based on symmetry.
- Note that, however, these studies are based on the first notion of computation in CA's, while Langton adopted the second one.
- A system might be capable of universal computations, but it might be not its "typical" behaviour, e.g. if it requires peculiar initial conditions

The "critical mind"

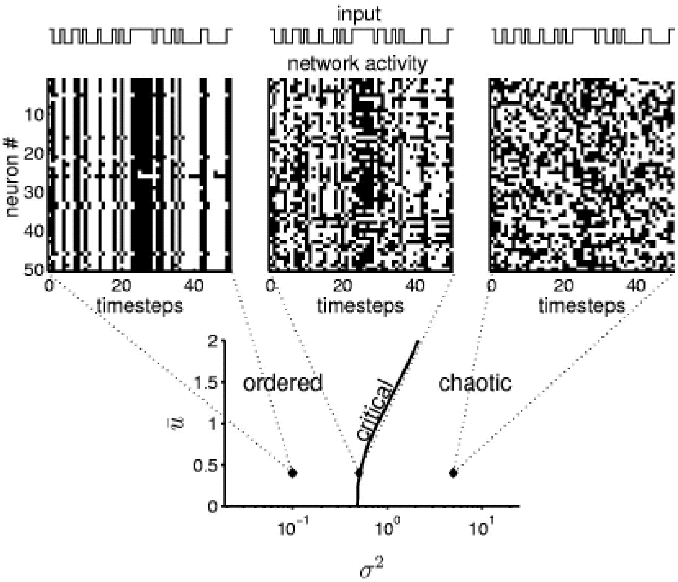
John M. Beggs in Nature Physics Vol.3 (2007), 834:

"Simulations of neural networks indicate that the critical point would be quite beneficial for information processing. Just as the correlation length and susceptibility are maximized in a ferromagnet at the critical point, the dynamic range, memory capacity and computational power are optimized in neural network simulations when they are tuned to the critical point."

The "critical mind"

- Per Bak proposed a SOC model for the brain ("minibrain"), where the input singnal corresponds to the driving input and the learning sequence represents the avalanche. In consists of a feedforward network with a winner takes it all dynamics. If the output is wrong all weigths on the active path are decreased.
- John M. Beggs (2003): power law distribution of the sizes of avalanches of network activity in slices of rat cortex
- Levina et al. (2007): SOC in a network of integrate and fire neurons with dynamical synapses
- Bertschinger et al. (2004): Showed in the framework of "liquid state machines" that certain neural networks exhibit the best memory and computational performance near the critical line of an order-disorder transition.

Network Dynamics



Computational Performance

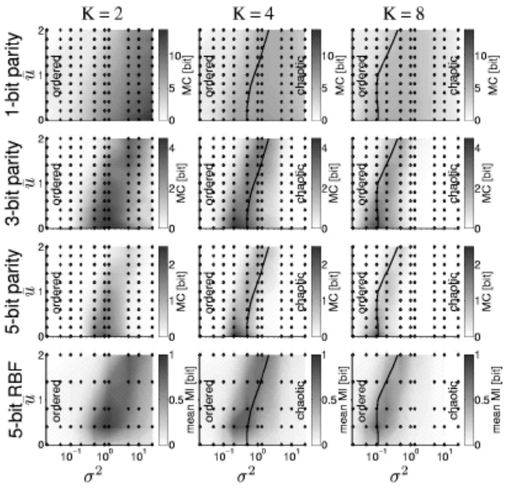


Figure 8: Performance of trained networks for various parameters and different tasks with increasing complexity. The performance (as measured in Figure 6B) is shown in dependence of the parameters σ^2 and $\bar{u}$ for $K = 2, 4, 8$ (left to right) and the 1-, 3-, and 5-bit parity task as well as for an average over 50 randomly drawn boolean functions (top to bottom).

Resumé

- "Edge of chaos" was and is an influential metaphor but in many contexts not a well defined concept.
- Long range and long term correlations are related to criticality on the one hand side and high complexity on the other hand side, thus connecting phase transitions and complexity.
- "Life at the edge of chaos"(Langton): There are some good arguments and also some interesting examples supporting the idea that transition regions between order and chaos are optimal for certain adaptive systems, but still no general theory exists.