Problem Sheet 4

1. Equiintegrability Let $\Omega \subset \mathbb{R}^n$ be bounded, $(f_j) \subset L^p(\Omega, \mathbb{R}^m)$, $p \in [1, \infty)$. Prove that the following statements are equivalent:

(i) $\lim_{R\to\infty} \sup_{j\in\mathbb{N}} \int_{|f_j|>R} |f_j|^p \,\mathrm{d}x = 0$

(ii) $\lim_{R\to\infty} \limsup_{j\to\infty} \int_{|f_j|>R} |f_j|^p \,\mathrm{d}x = 0$

(iii) for every $\varepsilon > 0$ there is $\delta > 0$ such that for all sets $B \subset \Omega$ Borel with $|B| < \delta$,

$$\sup_{j\in\mathbb{N}} \int_B |f_j|^p \,\mathrm{d}x < \varepsilon.$$

2. A weaker assumption suffices for generation Prove that a sequence of measurable maps $(V_j)\colon \Omega \to \mathbb{R}^N$ satisfying only the tightness condition

$$\lim_{h\to\infty} \sup_{j\in\mathbb{N}} |\{|V_j| \geq h\}| = 0,$$

also generates a Young measure in a suitable sense that you should define.

3. Example Take $\Omega = (0,1)$ and let $u_j(x) = \sin(2\pi j x)$ for $j \in \mathbb{N}$. Show that (u_j) generates the Young measure $(\nu) = (\nu)_x$ with

$$\nu_x = \frac{1}{\pi\sqrt{1-y^2}} \mathscr{L}^1_y(-1,1)$$

for almost every $x \in (0,1)$.