## Exercises 6

**1**. Let $G$ be a finite group with character table $X$ and $H$ a finite group with character table $Y$. Show that the Kronecker product $X \otimes Y$ is the character table of $G \times H$.

**2**. Let $\chi$ be a character of an abelian group $G$. Show that for any $b \in G$, we have

$$\chi(b) \sum_{a \in G} \chi(a) = \sum_{a \in G} \chi(a)$$

and use this to show that

$$\sum_{a \in G} \chi(a) = \begin{cases} |G| & \text{if } \chi \text{ is the character of the trivial representation,} \\ 0 & \text{otherwise} \end{cases}$$

**3**. Prove Lemma 1.11.

**4**. Let $a_n = 3^n \delta_n(3)$ be the maximum size of a subset $A \subseteq \mathbb{F}_3^n$ without nontrivial 3-term arithmetic progressions. Use Lemma 2.3 to show that $a_1 \leq 2$, $a_2 \leq 4$ and $a_3 \leq 9$. Show by example that equality holds in both cases.

**5**. Let $G$ be an abelian group and $A \subseteq G$ be a set without proper 3-APs. Show that the $k$-fold Cartesian product $A^k$ has no proper 3-APs in $G^k$.

**6**. Show that $a_n \geq (2.08)^n$ for sufficiently large $n$.