## Exercises 14

**1**. Recall that the character table of $S_4$ is:

| | (1) | (12) | (12)(34) | (123) | (1234) |
|---|---|---|---|---|---|
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | $-1$ | 1 | 1 | $-1$ |
| $\chi_3$ | 3 | 1 | $-1$ | 0 | $-1$ |
| $\chi_4$ | 3 | $-1$ | $-1$ | 0 | 1 |
| $\chi_5$ | 2 | 0 | 2 | $-1$ | 1 |

One way to find the 5 irreducible representations of $S_4$ would be to use the following equalities.

$$\begin{aligned} \chi_{\text{triv}} &= \chi_1 \\ \chi_{\text{alt}} &= \chi_2 \\ \chi_{\text{std}} &= \chi_1 + \chi_3 \\ \chi_4 &= \chi_2 \otimes \chi_3 \\ \chi_3 \otimes \chi_3 &= \chi_1 + \chi_3 + \chi_4 + \chi_5 \end{aligned}$$

Another way is to write down the Sprecht representation $S^\lambda$ for each Young diagram $\lambda$ with 4 boxes.

For each $i \in \{1, \ldots, 5\}$, determine the Young diagram $\lambda$ such that $S^\lambda$ has character $\chi_i$.

**2**. Consider the Young diagram $\lambda = (n-1, 1)$. Prove that the subrepresentation $S^\lambda$ of $M^\lambda$ is isomorphic to the subrepresentation $\{(x_1, \ldots, x_n) \in \mathbb{C}^n \mid x_1 + \ldots + x_n = 0\}$ of the standard representation.