Exercise Sheet 7

Please upload your solutions to URM or send them by email by Wednesday, December 14 at 14:00

Exercise 7.1 (5 points) Consider a complex torus $E = \mathbb{C}/\Lambda$. Since this is an additive group, we can consider the multiplication by n map, for an integer $n \in \mathbb{Z}$

$$[n]: E \to E, \qquad z \mapsto n \cdot z$$

a) Prove that $[n]$ is a nonconstant holomorphic map. What is its degree?

b) Show that $[n]$ has no ramification points.

Exercise 7.2 (5 points) Consider the upper half-plane $\mathcal{H} = \{\tau \in \mathbb{C} \mid \operatorname{Im} \tau > 0\}$ and the group

$$SL(2, \mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}$$

a) Show that $SL(2, \mathbb{Z})$ acts on the upper half-plane $\mathcal{H}$ by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \tau = \frac{a\tau + b}{c\tau + d}$$

b) Show that two complex tori $\mathbb{C}/\mathbb{Z} + \tau\mathbb{Z}$ and $\mathbb{C}/\mathbb{Z} + \mathbb{Z}\tau'$ are isomorphic if and only if there exists an element $\gamma \in SL(2, \mathbb{Z})$ such that

$$\tau = \gamma \cdot \tau'$$