Exercise Sheet 3

Please upload your solutions on URM or send them by email by Wednesday, November 16 at 14:00

Exercise 3.1 (4 points) Let $C = \{F(X, Y, Z) = 0\}$ be a projective plane curve and $P \in C$ be a point. Prove the projective Jacobi's criterion that we have stated in class:

$$\mathbb{T}_P C = \left\{ [X, Y, Z] \,|\, \frac{\partial F}{\partial X}(P) \cdot X + \frac{\partial F}{\partial Y}(P) \cdot Y + \frac{\partial F}{\partial Z}(P) \cdot Z = 0 \right\}$$

Exercise 2.2 (4 points) Let C be a reduced and irreducible plane curve (affine or projective). Show that C has finitely many singular points.

Exercise 2.3 (2 points)

a) Let $f(x) \in \mathbb{C}[x]$ be an univariate polynomial. Show that the curve $C = \{y^2 = f(x)\}$ is smooth if and only if f has no multiple root.

b) Show that the projective Fermat curve of degree d, given by $C_d = \{X^d + Y^d + Z^d = 0\}$ is smooth.