Exercise Sheet 8

Submit by: Monday, 20/06/22, 10 am

Exercise 8.1 Let $X \subseteq \mathbb{A}^n$ be an affine variety, and let $Y_1, Y_2 \subset X$ be irreducible, closed subsets, no-one contained in the other. Moreover, let $\tilde{X}$ be the blow-up of X at the ideal $I(Y_1) + I(Y_2)$.

Show that the strict transforms of Y_1 and Y_2 in $\tilde{X}$ are disjoint.

Exercise 8.2 Let $a = (1 : 0 : 0)$, $b = (0 : 1 : 0)$ and $c = (0 : 0 : 1)$ be the three coordinate points of $\mathbb{P}^2$, and let $U = \mathbb{P}^2 \smallsetminus \{a, b, c\}$. Consider the morphism

$$f : U \to \mathbb{P}^2, \quad (x_0 : x_1 : x_2) \to (x_1 x_2 : x_0 x_2 : x_0 x_1).$$

(a) Show that there is no morphism $\mathbb{P}^2 \to \mathbb{P}^2$ extending f.

(b) Let $\tilde{\mathbb{P}^2}$ be the blow-up of $\mathbb{P}^2$ at $\{a, b, c\}$. Show that f can be extended to an isomorphism $\tilde{f} : \tilde{\mathbb{P}^2} \to \tilde{\mathbb{P}^2}$.