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Algebraic Geometry
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Exercise Sheet 5

Submit by: Monday, 23/05/22, 10 am

Exercise 5.1 Observe that if Z is a variety and if $X, Y \subseteq Z$ are (open or closed) subvarieties, then $X \cap Y \cong (X \times Y) \cap \Delta_Z$ where the second intersection is taken inside $Z \times Z$. Use this to prove the following.

a) The intersection of two open affine subsets in a variety is again affine.

b) If $X, Y \subseteq \mathbb{A}^n$ are irreducible affine varieties, and $X \cap Y \neq \emptyset$, then every component of $X \cap Y$ has dimension at least $\dim X + \dim Y - n$.

Now let $X, Y \subseteq \mathbb{P}^n$ be two closed projective subvarieties, such that $\dim X + \dim Y \geq n$.

c) Prove that $X \cap Y$ is nonempty. [*Hint*: use point (b) on the affine cones $C(X), C(Y)$].

Exercise 5.2 Let $L_1, L_2 \subseteq \mathbb{P}^3$ be two disjoint lines an let $P \in \mathbb{P}^3$ be a point not lying on L_1 or L_2. Show that there is a unique line through P that intersects both L_1 and L_2.