Fachbereich Mathematik
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Summer Semester 2022, Set 2

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Algebraic Geometry

Submit by: Monday, 02/05/22, 10 am

Exercise 6 b. and 7 b. are an in class exercises. There is no need to hand the solution in.

Exercise 5: Compute the irreducible compontents of the affine variety

$$V(x_1 - x_2x_3, x_1x_3 - x_2^2) \subset \mathbb{A}^3_{\mathbb{C}}.$$

Hint compute the primary decomposition with SINGULAR to see what you should expect.

Exercise 6: Let X and Y be topological spaces and $\emptyset \neq U \subseteq X$.

a. Show, if $f : X \to Y$ is continous and X is irreducible, then $f(X)$ is irreducible.

b. Show, U is irreducible if and only if the closure $\overline{U}$ is irreducible.

Exercise 7:

a. Show that
$$X = \{A \in \mathrm{Mat}(2 \times 2, \mathbb{C}) \mid A \text{ is nilpotent}\}$$
is an irreducible affine variety in $\mathbb{A}^4_{\mathbb{C}} = \mathrm{Mat}(2 \times 2, \mathbb{C})$.

b. Show that
$$Y = \{A \in \mathrm{Mat}(2 \times 3, \mathbb{C}) \mid \mathrm{rank}(A) \leq 1\}$$
is an irreducible affine variety in $\mathbb{A}^6_{\mathbb{C}} = \mathrm{Mat}(2 \times 3, \mathbb{C})$.

What is the dimension of X and Y?

Hint: show first that X and Y are affine varieties, and use then Exercise 5 to show that a suitable open subset is irreducible.

Compute the Krull dimension of the defining ideals with SINGULAR to see what you should expect.

Exercise 8: Show, if X_1 and X_2 are two affine varieties in $\mathbb{A}^n_K$, then

$$I\left(\overline{X_1 \setminus X_2}\right) = I(X_1) : I(X_2).$$