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Algebraic Geometry
SS 2022

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Exercise Sheet 12

Submit by: Monday, 18/07/22, 10 am

Exercise 12.1 Consider the affine scheme $\mathbb{A}_k^2 = \mathrm{Spec}(k[x,y])$ over an algebraically closed field k.

(a) Describe the closed subscheme given as $X := \mathrm{Spec}(k[x,y]/\langle xy\rangle)$.

(b) Construct an isomorphism of schemes between $Y := \mathrm{Spec}(k[x,y]/\langle x-y\rangle)$ and the affine line $\mathbb{A}_k^1 = \mathrm{Spec}(k[z])$.

(c) Describe the continuous map underlying the morphism of schemes $f : X \to \mathbb{A}_k^1$, which is determined by the ring homomorphism

$$\begin{array}{cccc} f^* : & k[z] & \to & k[x,y]/\langle xy\rangle \\ & z & \mapsto & x+y \end{array}$$

Use (b) for a geometric interpretation.

(d) Determine the scheme-theoretic fibers over <u>all</u> points of $\mathbb{A}_k^1$.

Exercise 12.2 Recall that a point $P \in \mathbb{P}^n$ is by definition a 1-dimensional linear subspace $L_P \subset K^{n+1}$. The *tautological sheaf* $\mathcal{F}$ on $\mathbb{P}^n$ is defined by

$$\mathcal{F}(U) := \{\phi : U \to K^{n+1} \text{ morphism with } \phi(P) \in L_P \text{ for all closed points } P \in U\}$$

for all open subsets $U \subseteq \mathbb{P}^n$.

Prove that $\mathcal{F}$ is isomorphic to the twisting sheaf $\mathcal{O}_{\mathbb{P}^n}(-1)$.

Exercise 12.3 Find a number $d \in \mathbb{Z}$ and morphisms of sheaves α and β such that the sequence

$$0 \longrightarrow \mathcal{O}_{\mathbb{P}^1} \xrightarrow{\alpha} \mathcal{O}_{\mathbb{P}^1}(1) \oplus \mathcal{O}_{\mathbb{P}^1}(1) \xrightarrow{\beta} \mathcal{O}_{\mathbb{P}^1}(d) \longrightarrow 0$$

is exact on $\mathbb{P}^1$.