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Algebraic Geometry
SS 2022

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Exercise Sheet 10

Submit by: Monday, 04/07/22, 10 am

Exercise 10.1 Find an example of the following, or prove that it does not exist:

(a) an irreducible affine scheme $\operatorname{Spec}(R)$ such that R is not an integral domain;

(b) two affine schemes $\operatorname{Spec}(R)$ and $\operatorname{Spec}(S)$ with R ≤ S and $\dim(\operatorname{Spec}(R)) > \dim(\operatorname{Spec}(S))$;

(c) a point of $\operatorname{Spec}(\mathbb{R}[x_1, x_2]/\langle x_1^2 + x_2^2 + 1\rangle)$ with residue field $\mathbb{R}$;

(d) an affine scheme of dimension 1 with exactly two points.

Exercise 10.2

(a) Let X be a topological space and let $S \subset X$ be a subset. Show that S is dense in X if and only if any non-empty open subset of X contains a point of S.

(b) Let $R = A(X)$ be the coordinate ring of an affine variety X over an algebraically closed field k. Show that the set of all closed points is dense in $\operatorname{Spec}(R)$.

(c) Show by example that on a general affine scheme the set of all closed points needs not be dense.

Exercise 10.3 Show that any affine scheme is quasi-compact, i.e. any open covering of $\operatorname{Spec}(R)$ contains a finite subcover.