Fachbereich Mathematik
Daniele Agostini Summer Semester 2022, Set 1
Thomas Markwig

Algebraic Geometry

Submit by: Monday, 25/04/22, 10 am

Exercise 1: Let $X \subset \mathbb{A}^3_K$ be the union of the three coordinate axes.

a. Compute generators for the ideal $I(X)$.

b. Show that $I(X)$ cannot be generated by fewer than three elements.

Hint: in part b. one can use Nakayama's lemma after localising.

Exercise 2: Prove that every affine variety $X \subset \mathbb{A}^n_K$ consisting of only finitely many points can be written as the zero locus of n polynomials.

Hint: use interpolation and consider first the case that the x_1-coordinates of the points are different.

Exercise 3: [Irreducible components]
Let X be a topological space and Y a non-empty irreducible subspace of X. Show, that there is a maximal irreducible subspace Y' in X containing Y, i.e. there is a $Y' \subseteq X$ irreducible such that $Y \subseteq Y'$ and for all irreducible $Y'' \subseteq X$ with $Y' \subseteq Y''$ we have $Y' = Y''$.

Exercise 4: Let $X \subseteq \mathbb{A}^n_K$ and $Y \subseteq \mathbb{A}^m_K$ be two affine varieties and consider their product variety $X \times Y$ in $\mathbb{A}^{n+m}_K$.

a. Show, for every $b \in Y$ the map $i_b : X \longrightarrow X \times Y : x \mapsto (x, b)$ is continous.

b. Show that the projections from $X \times Y$ to the X or Y map open sets to open sets.

c. Show, if X and Y are irreducible, then so ist $X \times Y$.