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Summary. We consider the theoretical determination of firing rates in some biological neural networks that consist of synaptically connected excitatory and inhibitory elements. A self-consistent argument is employed to obtain equations satisfied by the moments of the firing times of the various cells in the network. We first present results for networks composed of leaky integrate-and-fire model neurons in the case of impulsive currents representing synaptic inputs and an imposed threshold for firing. Solving a differential-difference equation with specified boundary conditions yields an estimate of the mean interspike interval of neurons in the network. We graphically demonstrate that there may be a critical number of connections $n = n_c$ such that for $n < n_c$ there is no nontrivial solution whereas for $n > n_c$ there are three solutions. Of these, one is at baseline activity, one is unstable and the other is asymptotically stable. Simulation results are reported which demonstrate that sustained activity is possible even without external afferent input and that the analytical method may yield accurate estimates of the firing rate. We also consider a network of generalized Hodgkin-Huxley model neurons. Assuming a voltage threshold, which is a useful representation for slowly firing such nerve cells, a functional differential equation is obtained whose solution affords an estimate of the mean network firing rate. Related equations enable one to estimate the second- and higher-order moments of the interspike interval.

1.1 Introduction

Investigations and theories of information processing in the nervous system have recently addressed the role of stochasticity in the activity of both single neurons [1–4] and neural networks [5–8]. One source of variability that occurs in cortical (and other) neurons is the “spontaneous” spiking activity in the absence of deliberate or known external input. The role of such spontaneous activity in relation to the processing of afferent input is not yet fully understood. However, spontaneous activity is now not considered to be simply noisy

background activity. According to [9], the spontaneous activity of many brain neuronal networks determines the nature of the response of the brain to external stimulation and may contribute to the quality of perception. In visual information processing, it has been found that spontaneous activity does indeed create extra variability in nervous system responses to visual stimuli [10]. Furthermore, some or all such spontaneous activity has been hypothesized to play an important role in visual information processing as it tends to provide a substrate for modulation by afferent input.

Spontaneous neural activity arises because the brain contains a large number of neurons with very dense patterns of connections between cells. Sporadic or sustained random inputs from the thalamus may occur at various parts of the cerebral cortex [9]. Small trains or bursts of activity in one region, possibly due to spontaneous transmitter release at synaptic terminals, are propagated to other regions and may circulate in recurrent loops. Some of the details of brain connection patterns have recently been described in detail for neurons of the mammalian neocortex [11]. However, in most studies, as for example in [5–8], and the current chapter, the precise details of the connections are omitted in favour of randomly assigned connections or fully connected networks. In a sense, spontaneous neural activity represents the simplest electrophysiological state or ground state for populations of neurons in the living brain, even though the underlying circuits are possibly as complex as for networks involved with perception or cognition. It is clear that by virtue of the complexity of the spatial and temporal aspects of the dynamics of individual neurons as well as the enormous complexity of the underlying biochemical processes, there are difficulties with analytical approaches to finding the quantitative features of spiking or nonspiking activity in neural networks with any degree of realism for the physiological properties of the composite single neurons. However, they may be useful for networks of artificial neurons, which are not usually endowed with a complex biochemical environment. Analytical results concerning dynamical neuronal behaviour are few even for single neurons [2, 12–15] and very sparse for neural networks composed of nonlinear elements, so that efficient simulation and other techniques have been developed for networks with stochastic dynamics [7, 16].

In many network models the neurons are considered to be weakly connected so that they are treated as independent and a mean field approach is adopted. It is then possible to use solutions of a single-neuron Fokker-Planck (or Kolmogorov) equation for the neuronal depolarization [5, 12–15], with suitable boundary conditions, to provide an estimate of the fraction of cells in the network at various levels of excitation. Background noise, usually represented by Poisson processes or white noise, has been found to play an important role in the propagation of signals through model networks [6] and in understanding the behaviour of cortical and subcortical neurons involved in cognitive information processes such as working memory [17–21].

It is of interest to attempt to determine as far as possible with analytical techniques the relationships between the microscopic physiological variables

associated with single nerve cells, their patterns of connectivity, and global spiking activity. Such relationships are expected to assist in casting light on computational aspects of neuronal information processing and cognitive activity. Here we describe a method for analytically estimating the firing rates of neurons in stochastic networks of an arbitrary number of elements using Markov process theory. We consider, in section 1.2, networks of generalized integrate-and-fire neurons and present some exact results for networks of excitatory neurons. In section 1.3 we state corresponding results for networks composed of neurons with generalized Hodgkin-Huxley dynamics. In order that the theoretical estimates are as accurate as possible, our attention is focused mainly on slowly firing networks whose spiking is in singlets. We explore the relations between single neuron dynamics, network size and the nature of sustainable spontaneous activity.

1.2 Networks of Generalized Leaky Integrate-and-Fire (LIF) Neurons

Let the electrical depolarization from rest for the k th cell in a network of n neurons be $V_k(t)$ at time t . In general, in the LIF scheme, the dynamical equations for the network can be written

$$dV_k = f(V_k)dt + \sum_{j=1}^n a_{jk}dN_j(\lambda_j; t - d_{jk}) + \epsilon_k dN_k^{ext}, k = 1, 2, \dots, n, \quad (1.1)$$

where a_{jk} (element of the matrix A) is the strength of the connection from neuron j to neuron k , which determines the magnitude of the postsynaptic potential elicited when this connection is activated (it is assumed that $a_{jj} = 0$ for all j). The threshold for action potential generation in the k th cell is set at θ_k , here assumed to be constant. $N_j(\lambda_j; t)$ is the number of spikes emitted by neuron j in the time interval $(0, t]$; the parameter λ_j represents the mean rate of spiking so that as usual an approximation $1/\lambda_j$ can be interpreted as the mean time between spikes from neuron j . The quantities d_{jk} are the time delays between emission of a spike by cell j and the appearance of a postsynaptic potential in cell k . The process N_k^{ext} is an external Poisson input which produces synaptic potentials of magnitude ϵ_k . According to (1.1), the input currents are delta functions so that the detailed temporal structure of postsynaptic potentials is here ignored in a first approximation. This structure can easily be incorporated by adding subsidiary variables, but this would make the theory much more complicated and so is omitted.

1.2.1 A Network of Excitatory and Inhibitory Neurons

Suppose there are n_E excitatory (type E) and n_I inhibitory (type I) cells, whose membrane potentials in the absence of synaptic activation satisfy $\dot{V}_E = f_E(V_E)$ and $\dot{V}_I = f_I(V_I)$, respectively. The postsynaptic potential amplitudes are the nonnegative quantities a_{EE}, a_{IE}, a_{EI} and a_{II} , where the first subscript refers to the presynaptic cell type and the second to the postsynaptic cell type. The corresponding numbers of connections for individual cells are n_{EE}, n_{IE}, n_{EI} and n_{II} , respectively. Thus each E -cell receives a total of $n_{EE} + n_{IE}$ synaptic inputs and each I -cell receives $n_{EI} + n_{II}$ such inputs. Since each E -cell has effectively the same input, its membrane potential satisfies

$$dV_E = f_E(V_E)dt + a_{EE}dN_{EE}(n_{EE}\lambda_E; t) - a_{IE}dN_{IE}(n_{IE}\lambda_I; t), \quad (1.2)$$

whereas for each I -cell,

$$dV_I = f_I(V_I)dt + a_{EI}dN_{EI}(n_{EI}\lambda_E; t) - a_{II}dN_{II}(n_{II}\lambda_I; t), \quad (1.3)$$

where N_{EE}, N_{IE} are the pooled excitatory and inhibitory input point processes for E -cells, and N_{EI}, N_{II} are the corresponding processes for the I -cells. Here we do not include external afferent input (or background noise) which can be taken into account as in subsection 2.2. Letting the thresholds of the two kinds of cell be θ_E and θ_I , we then have the following result, where now refractory periods are included. The proof of this follows by considering the total input frequencies for the component neurons and their respective expected time intervals for their membrane potentials to reach threshold.

Theorem

If $\frac{1}{\lambda_E} + t_{R,E}$ is the mean time interval between spikes in an E -cell and $\frac{1}{\lambda_I} + t_{R,I}$ is the mean time interval between spikes in an I -cell of the network, then these quantities may be estimated implicitly by solving the simultaneous differential-difference equations

$$\begin{aligned} f_E(v) \frac{dF_E}{dv} + n_{EE}\Lambda_E F_E(v + a_{EE}) + n_{IE}\Lambda_I F_E(v - a_{IE}) \\ - (n_{EE}\Lambda_E + n_{IE}\Lambda_I) F_E(v) = -1, \quad v < \theta_E, \end{aligned} \quad (1.4)$$

$$\begin{aligned} f_I(v) \frac{dF_I}{dv} + n_{EI}\Lambda_E F_I(v + a_{EI}) + n_{II}\Lambda_I F_I(v - a_{II}) \\ - (n_{EI}\Lambda_E + n_{II}\Lambda_I) F_I(v) = -1, \quad v < \theta_I, \end{aligned} \quad (1.5)$$

with boundary conditions $F_E(v) = 0, v \geq \theta_E, F_I(v) = 0, v \geq \theta_I$ and $F_E(0) = \frac{1}{\lambda_E}, F_I(0) = \frac{1}{\lambda_I}$, provided such solutions exist.

Here $F_E(v)$ and $F_I(v)$ are the mean times for the potentials V_E and V_I to reach thresholds from initial values v , and we have put

$$\Lambda_E = \left(\frac{1}{\lambda_E} + t_{R,E} \right)^{-1} \quad \Lambda_I = \left(\frac{1}{\lambda_I} + t_{R,I} \right)^{-1}$$

where $t_{R,E}$ and $t_{R,I}$ are the refractory periods of the excitatory and inhibitory cells, respectively. In general, in the search for such solutions, one may insist that F_E and F_I vanish for $v < v_E < 0$ and $v < v_I < 0$, respectively, and then let v_E and $v_I \rightarrow -\infty$ to ensure that the thresholds for action potentials are attained.

Equations (1.4) and (1.5) are difficult to solve exactly but may be solved numerically or approximately via simulation of the corresponding stochastic differential equations. In the simpler situation of a uniform network where each cell has the same membrane potential dynamics, including thresholds θ for action potentials, and the same numbers of excitatory, n_E , and inhibitory, n_I , inputs, with the same amplitudes a_E and a_I for the synaptic potentials, the same refractory periods t_R , and all cells fire at the same mean rate λ , (1.4) and (1.5) reduce to the single differential-difference equation

$$f(v) \frac{dF}{dv} + \Lambda [n_E F(v+a_E) + n_I F(v-a_I)] - (n_E + n_I) \Lambda F(v) = -1, \quad v < \theta, \quad (1.6)$$

where $\Lambda = (1/\lambda + t_R)^{-1}$; (1.6) is solved with the constraints $F(v) = 0, v \geq \theta$ and $F(0) = \frac{1}{\lambda}$. Numerical solutions of (1.6) have been obtained in [28]. Diffusion approximations may be employed to obtain approximate estimates.

1.2.2 A Network of Only Excitatory Cells

Suppose that all connections are excitatory so that $a_{jk} \geq 0$ and each cell *receives* the same number of inputs, n_E . The network may or may not be fully connected, as long as each neuron receives synaptic input from n_E other cells. The number of inputs is thus $n_E \leq n - 1$, with equality applying in the case of a fully connected network. It is feasible that there are neurons that are not presynaptic to any other cells, although this is unlikely in the mammalian central nervous system. To simplify, we ignore the transmission time intervals d_{jk} and assume further that all nonzero $a_{jk} = a_E$, which quantities may in fact be random but are here assumed to be deterministic or at their mean values. We assume also that all cells have equal thresholds $\theta_j = \theta, j = 1 \dots, n$ and the same subthreshold dynamics described by the function f . The network is then described by the stochastic equations

$$dV_k = f(V_k) dt + a_E \sum_{j=1}^n dN_j(\lambda_j; t - d_{jk}) + \epsilon_k dN_k^{ext}, \quad k = 1, 2, \dots, n. \quad (1.7)$$

Now each cell receives excitatory postsynaptic potentials according to n_E similar pooled point processes with the same mean rates, which we set at λ_E , and their amplitudes are all a_E so the generic depolarization V of each neuron satisfies

$$dV = f(V)dt + a_E dN(n_E \lambda_E; t) + \epsilon dN^{ext}, V < \theta, V(0) = v < \theta. \quad (1.8)$$

In general we have the following result whose proof is immediate from the theory of mean exit times of Markov processes [27] and the requirement that the network input frequency in a network of identical neurons each of which receives n_E inputs is n_E times the output frequency of each cell. Here $F(v)$ is the mean exit time of V from $(0, \theta)$ for an initial value $V(0) = v$.

Theorem

If $\frac{1}{\lambda_E}$ is the mean time interval between spikes in a neuron of the network described by (1.7), then it may be estimated implicitly by solving the differential-difference equation

$$f(v) \frac{dF}{dv} + n_E \lambda_E [F(v + a_E) - F(v)] + \lambda^{ext} [F(v + \epsilon) - F(v)] = -1, \quad v \in (0, \theta), \quad (1.9)$$

with boundary conditions $F(v) = 0, v \geq \theta$ and $F(0^+) = \frac{1}{\lambda_E}$, provided such a solution exists.

However, with the above kind of single-neuron model, the time interval between spikes may have to take into account a refractory period, t_R , of order a few msec, which may be significant. In this case (1.9) becomes

$$f(v) \frac{dF}{dv} + \frac{n_E}{\lambda_E^{-1} + t_R} [F(v + a_E) - F(v)] + \lambda^{ext} [F(v + \epsilon) - F(v)] = -1, \quad v < \theta, \quad (1.10)$$

but the second boundary condition is still $F(0^+) = \frac{1}{\lambda_E}$.

1.2.2.1 Graphical and Numerical Methods of Solution

We illustrate graphically that a nontrivial solution of (1.10) may fail to exist or there may be three solutions, one of which may correspond at least approximately to a stable connected network with nonzero firing rates. To do this we consider a network of cells whose common frequency transfer characteristic, including the effect of an external input and a refractory period, is known and

is as depicted by the blue curve in Figure 6.1. It is assumed that all postsynaptic potentials are the same size and that background activity, represented by the terms $\epsilon_k dN_k^{ext}$, drives each cell to fire at rate f_b , which may be zero. For a cell to be a member of the given network, if each cell fires at rate f_o , the total input frequency to each cell must be nf_o above background, where n is the number of cells which connect to any given cell. Hence it is required that the output frequency is related to input frequency above background f_i by $f_i = nf_o$ or $f_o = f_i/n$. Such straight lines are drawn in Figure 6.1.

For the straight line with the largest slope and hence the smallest value of n , labeled case A, there is no intersection with the frequency transfer function of the neuron so that theoretically no sustainable firing above background is possible in a network with this number of connections to each cell. For a critical value n_c of the number of connections, $f_o = \frac{1}{n_c} f_i$, labeled B, the line is tangent to the frequency transfer curve. For $n > n_c$, as typified by the line labeled C, there are three solutions, one being at $f_i = 0$ (external input only) and output frequency f_b . Of the other two, P_2 is unstable whereas the asymptotically stable point P_1 corresponds to a possible stable network frequency. Furthermore, if a network has an observed firing rate f_s , the number of neurons in the network or the number of connections to each neuron could be estimated by ascertaining where $f_o = nf_s$ intersects the transfer curve. The latter could be obtained empirically. The reciprocal of the slope of the line from the point (λ^{ext}, f_b) to the stable point of intersection gives an estimate of n .

Exact analytical solutions have not been found for (1.9) for $\theta > 3$, but numerical methods are available [22]. However, the numerical methods are not simple to implement so that simulations of network activity are useful and have been performed for a large range of network sizes and for various parameter values. A problem often encountered is that neurons tend to fire synchronously in groups which makes the analytical methods described in this chapter not applicable because of the underlying assumption of randomness. It has been found that it is unusual to find sustained network activity in the absence of an external afferent drive, ϵdN^{ext} . One example is shown in Figure 6.2. In this simulation there were 1000 LIF neurons with on average 400 connections per cell. The internal excitatory synaptic potential amplitude was 0.04 mV and the external input to each cell had a rate $\lambda^{ext} = 500$ per sec with amplitude $\epsilon = 1$ mV. The time step used was $\Delta t = 0.00005$ sec. The afferent input was terminated at the 200th time step, and yet activity is seen after a short decline, to rise spontaneously to an apparently steady state. Several checks on the validity of the analytical method were performed. For example, for a fully connected excitatory network of 50 neurons, with $\epsilon = 1$ mV, $\lambda^{ext} = 500$ per sec, $a_E = 0.2$ mV, threshold 15mV, and time constant 20 msec, the network individual neuronal frequency obtained by simulation was 7.0 per sec whereas that predicted by the analytical method was 8.4 per sec.

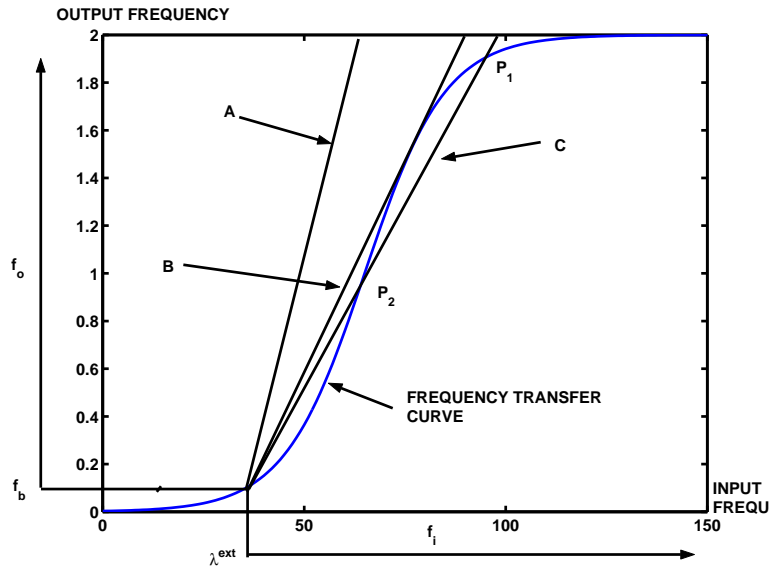


Fig. 1.1. Output frequency is plotted against input excitatory frequency. The frequency transfer characteristic of cells in the network is indicated by the blue curve. f_o is the output frequency above f_b , which corresponds to an external input frequency of λ^{ext} . f_i is input frequency above λ^{ext} . The straight lines A, B and C represent cases where the output frequency is a given fraction of the network input frequency for each cell, so $f_o = f_i/n$ where n is the number of cells which connect to each cell. In case B, $f_o = f_i/n_c$. For more details, see the text.

1.2.3 Diffusion Approximations

An alternative approach is to use smoothed versions of the network voltage processes called diffusion approximations. For such processes it is relatively easy to find the transfer characteristic not only for neurons of purely excitatory networks but also those including both excitation and inhibition. The validity of such an approach depends on parameter values [23]. Further details of such approximations, which have computational advantages, will be reported elsewhere.

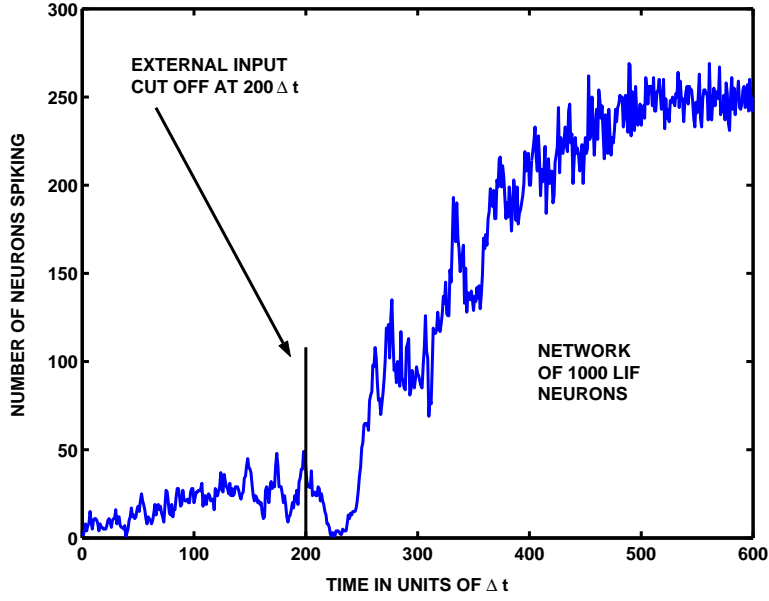


Fig. 1.2. Simulated activity in a network of 1000 LIF model neurons. The number of neurons firing is plotted against time. The external afferent input to each cell was terminated at $200 \Delta t$. Note the sustained activity in the absence of afferent input. The parameter values are given in the text.

1.3 A Network of Neurons with Generalized Hodgkin-Huxley Dynamics

As is well known, the original Hodgkin-Huxley model [24] describes subthreshold activity and action potential generation in squid axon and contains only sodium, potassium and leak currents. However, many types of ion channels with distinct roles and dynamics [25, 26] have been found in, for example, pyramidal cells, so it is useful to generalize the Hodgkin-Huxley system. Towards such a generalization we consider $M + 1$ neurons described by the dynamical system

$$C_i \frac{dU_i}{dt} = - \sum_{k=1}^n \bar{g}_{ki} (U_i - V_{ki}) \prod_{j=1}^{p_k} \gamma_{kj}^{m_{kj}} + I_i,$$

$$\frac{d\gamma_{kji}}{dt} = \alpha_{kj}(U_i)(1 - \gamma_{kji}) - \beta_{kj}(U_i)\gamma_{kji},$$

where $i = 1, \dots, M + 1$, V_{ki} = Nernst potential for the k th ion species for the i th neuron, the maximal conductances are \bar{g}_{ki} , there are p_k gating variables

associated with the k th ion species and I_i is the total input current for the i th neuron. (Note that $n = 3$ for the Hodgkin-Huxley system.)

In a network of connected such neurons we suppose that

$$C_i \frac{dU_i}{dt} = - \sum_{k=1}^n \bar{g}_{ki} (U_i - V_{ki}) \prod_{j=1}^{p_k} \gamma_{kj}^{m_{kj}} + \epsilon_i dN_i^{ext} + \sum_l a_{li} dN_l(\lambda_l; t - \Delta_{li}),$$

where $i, l = 1, \dots, M + 1$, N_l is the output process describing the spikes of the l th neuron and where $\{a_{li}, i, l = 1, \dots, M + 1\}$ is the connection matrix, Δ_{li} is a time delay and $\epsilon_i dN_i^{ext}$ is an external input to cell i .

1.3.1 Network Firing Rate

Let now N be an (approximating) Poisson process representing the input train for each cell. Then for a generic cell, ignoring the time delays of transmission, with each cell firing at rate λ , and assuming a fully connected network,

$$C \frac{dU}{dt} = - \sum_{k=1}^n \bar{g}_k (U - V_k) \prod_{j=1}^{p_k} \gamma_{kj}^{m_{kj}} + \epsilon dN^{ext} + a dN(M\lambda; t),$$

where all nonzero elements of the connection matrix are set at $a > 0$ so that all synapses are excitatory.

Let the infinitesimal (Markov) operator associated with the neuron's dynamics be \mathcal{L}_{neuron} . For example, for the usual leaky integrate-and-fire model with

$$\frac{dV}{dt} = \alpha(V) + \text{synaptic input}$$

we have

$$\mathcal{L}_{neuron} f(x) = \alpha(x) \frac{df}{dx}.$$

Then let $F(u, \mathbf{v})$ be the first exit time of U to suprathreshold values with an initial value (u, \mathbf{v}) where u is the initial voltage and \mathbf{v} contains the initial values of all gating variables (m, n and h for Hodgkin-Huxley). Then, assuming that the spiking of each cell is not too rapid and occurs in distinct singlets, and that the synaptic potentials are small relative to threshold such that synchronization is not predominant, we have the following result, which states that the network firing rate can then be *estimated*, using the single neuron operator, \mathcal{L}_{neuron} , as the solution of a (very complicated) differential-difference equation with the stated boundary conditions. Similar equations hold for the higher moments of the firing time.

Theorem

The network firing rate λ can be estimated from the solution of the functional differential equation

$$\mathcal{L}_{neuron}F(u, \mathbf{v}) + M\lambda[F(u+a, \mathbf{v}) - F(u, \mathbf{v})] + \lambda^{ext}[F(u+\epsilon, \mathbf{v}) - F(u, \mathbf{v})] = -1,$$

with the appropriate boundary conditions and the additional constraint

$$F(0, \mathbf{v}_0) = \frac{1}{\lambda},$$

where \mathbf{v}_0 contains the chosen initial values of the subsidiary variables.

Proof

The theorem follows immediately from standard first exit-time theory for discontinuous Markov processes [27], assuming that each neuron fires at rate λ and receives excitation from other cells in the network at rate $M\lambda$.

1.4 Discussion

We have outlined methods for determining the statistics of neuronal spiking activity in networks of neurons with general physiological properties (sub-threshold dynamics, threshold and refractory period) in cases where random activity can be sustained. Solutions of functional differential equations yield estimates of the firing rates and higher moments of the interspike interval. These solutions can be found by graphical, numerical or simulation methods. We have concentrated on networks whose neuronal spike trains are roughly Poisson and given results for excitatory networks. We demonstrated graphically that there may be three solutions where $f_o = f_i/n_E$ intersects the frequency transfer curve of each neuronal element and that one of these was stable and gave the frequency of spontaneous activity. With realistic parameter values it is thus possible to estimate the average number of connections to each cell. This approach may provide insight into the factors controlling spontaneous activity of neurons in the mammalian and other nervous systems. Networks with small excitatory postsynaptic potential amplitudes and those with excitatory and inhibitory connections can sometimes be analyzed more easily with diffusion approximations. The theory is accurate only for networks in which synchronization is unlikely as this tends to invalidate the assumption of continued randomness in individual neuronal spike trains. This was amply demonstrated in simulations of networks containing up to 1000 neurons. Additional sources of randomness, especially those which can be considered to

be of a perturbative nature and tend to decrease the chance of synchronization can easily be included in the analytical framework presented here. Such perturbative noise has been included, and found to play an essential role in maintaining network activity in studies of both a theoretical [6] and more applied nature [17, 18].

References

1. Badoual M, Rudolph M, Piwkowska Z, Destexhe A, Bal T (2005) High discharge variability in neurons driven by current noise. *Neurocomputing* 65–66: 493–498.
2. Feng JF, Tuckwell HC (2003) Optimal control of neural activity. *Phys Rev Lett* 91: 018101-5.
3. Jolivet R, Rauch A, Lüscher HR, Gerstner W (2006) Predicting spike timing of neocortical pyramidal cells by simple threshold models. *J Comp Neurosci* 21: 35–49.
4. Heil P (2004) First-spike latency of auditory neurons revisited. *Curr Opin Neurobiol* 14: 461–467.
5. Mattia M, Del Giudice P (2003) A distribution of spike transmission delays affects the stability of interacting spiking neurons. *Scientiae Mathematicae Japonicae* 18: 335–342.
6. Van Rossum MCW, Turrigiano GG, Nelson SB (2002) Fast propagation of firing rates through layered networks of noisy neurons. *J Neurosci* 22: 1956–1966.
7. Brunel N, Wang XJ (2003) What determines the frequency of fast network oscillations with irregular neural discharges? *J Neurophysiol* 90: 415–430.
8. Shiino M, Yamana M (2004) Statistical mechanics of stochastic neural networks. *Phys Rev E* 69: 011904.
9. Steriade M (2000) Corticothalamic resonance, states of vigilance and mentation. *Neuroscience* 101: 243–276.
10. Fiser J, Chiu C, Weliky M (2004) Small modulation of ongoing cortical dynamics by sensory input during natural vision. *Nature* 431, 573–578.
11. Douglas RJ, Martin KAC (2004) Neuronal circuits of the neocortex. *Ann Rev Neurosci* 27: 419–451.
12. Tuckwell HC, Wan FYM, Rospars JP (2002) A spatial stochastic neuronal model with Ornstein-Uhlenbeck input current. *Biol Cybernetics* 86: 137–145.
13. Burkitt AN (2006) A review of the integrate-and-fire neuron model: I. Homogeneous synaptic input. *Biol Cybernetics* 95: 1–19.
14. Holden AV (1976) *Models of the Stochastic Activity of Neurones*. Springer-Verlag, Berlin.
15. Gerstner W, Kistler WM (2002) *Spiking Neuron Models*. Cambridge University Press, Cambridge.
16. Rodriguez R, Tuckwell HC (1996). Statistical properties of stochastic nonlinear dynamical models of single neurons and neural networks. *Phys Rev E* 54: 5585–5590.
17. Durstewitz D, Seamans JK, Sejnowski TJ (2000) Neurocomputational models of working memory. *Nature Neurosci* 3 Supp: 1184–1191.

18. Durstewitz D, Seamans JK, Sejnowski TJ (2000) Dopamine-mediated stabilization of delay-period activity in a network model of prefrontal cortex. *J Neurophysiol* 83: 1733–1750.
19. Berns GS, Sejnowski TJ (1998) A computational model of how the basal ganglia produce sequences. *J Cogn Neurosci* 10: 108–121.
20. Durstewitz D, Seamans JK (2002) The computational role of dopamine D1 receptors in working memory. *Neural Networks* 15: 561–572.
21. Amit DJ, Brunel N (1997) Model of global spontaneous activity and local structured activity during delay periods in the cerebral cortex. *Cerebral Cortex* 7: 237–252.
22. Tuckwell HC, Richter W (1978) Neuronal interspike time distributions and the estimation of neurophysiological and neuroanatomical parameters. *J Theor Biol* 71: 167–183.
23. Tuckwell HC, Cope DK (1980) The accuracy of neuronal interspike times calculated from a diffusion approximation. *J Theor Biol* 83: 377–387.
24. Hodgkin AL, Huxley AF (1952) A quantitative description of membrane current and its application to conduction and excitation in nerve. *J Physiol* 117: 500–544.
25. Mainen ZF, Joerges J, Huguenard JR, Sejnowski TJ (1995) A model of spike initiation in neocortical pyramidal neurons. *Neuron* 15: 1427–1439.
26. Migliore M, Hoffman DA, Magee JC, Johnston D (2004) Role of an A-type K⁺ conductance in the back-propagation of action potentials in the dendrites of hippocampal pyramidal neurons. *J Comp Neurosci* 7: 5–15.
27. Tuckwell HC (1976) On the first-exit time problem for temporally homogeneous Markov processes. *J Appl Prob* 13: 39–48.
28. Cope DK, Tuckwell HC (1979) Firing rates of neurons with random excitation and inhibition. *J Theor Biol* 80: 1–14.

