formation which fits galactic observations quite well. The amplitude of the CDM density fluctuations had to be biased so it would yield fluctuations in mass which were a factor $1/b$ times smaller than fluctuations in galactic number density (evaluated on a scale of $8H^{-1}$ Mpc, where H is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}, and b is bias parameter). N-body simulations[1] of a universe filled with cold particles showed that the best fit between CDM overdensities and galaxies was obtained when $1.7 < b < 3.0$. But with this level of bias, CDM underpredicts the amount of large-scale structure characterized by such quantities as cluster correlations, bulk flows of galaxies, and galactic concentrations observed in redshift surveys. Furthermore, it predicts a quadrupole signal of $4.7 \times 10^{-6}/b$, which is significantly below the measured COBE value[2] of $6.1 \pm 1.5 \times 10^{-6}$.

By 1983, grand unified theories of particle physics had been constructed[3,4] in which both types of dark matter (cold plus hot dark matter, or CPHDM) arose naturally with comparable cosmological densities. The formation of large-scale structure with CPHDM was sketched out[5], and there were more precise calculations of the power spectra. It was emphasized that CPHDM gave more large-scale power and less small-scale power than a similarly normalized CDM model.

The CPHDM models were also more consistent[6,7] with bulk flow velocities. A consequence of their enhanced large-scale power was that the prediction[6,7] for the quadrupole moment was somewhat bigger than for the CDM model. The HDM fraction and the bias parameter were determined from comparisons of CPHDM models with observations[8]. A model with 1/4 HDM and 3/4 CDM and a realistic bias parameter of about b=1.1–1.4 gives a good fit to cluster number densities, cluster correlations, bulk flows of galaxies, and the Infrared Astronomy Satellite survey power spectrum. We recently showed[9] that CPHDM could account for the estimated co-moving density of high redshift quasars out to a redshift of 6. For this model, the quadrupole is predicted to be $7.8 \times 10^{-6}/b$, which for b=1.3 is in excellent agreement with the COBE value.

In short, CPHDM models with roughly 1/4 HDM and 3/4 CDM score well on every test applied to them. Indeed, they do very well at explaining galaxy formation[10], the traditional stronghold of CDM models. Perhaps the most crucial new test for the CPHDM model will come from measurements of the anisotropies expected on smaller angular scales. The predicted values of the anisotropy (with H=0.5 and 5% baryons) on 1° and 2.1° are $3.4 \times 10^{-5}/b$ and $2.3 \times 10^{-5}/b$, respectively. For b=1.3, the 1° prediction is less than a factor of two below the current upper limit[11].

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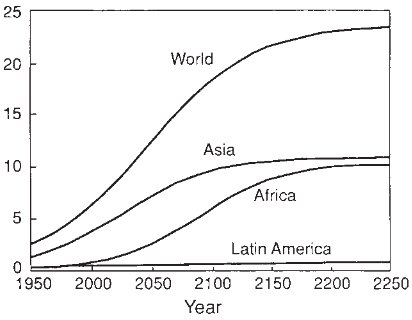
World population

SIR — The United Nations makes predictions of the human population of the Earth and continental regions[1] by the component method[2]. This requires a large number of parameters, such as birth and death rates within various age groups. An alternative approach is to use a model based on a differential equation involving only a few parameters. Three well-known such models are the malthusian, Verhulst's logistic and the gompertzian. The malthusian has been used to estimate previous global populations[3], but the logistic and gompertzian have the desirable property of limiting the final population size. We have examined human population data in relation to these three models.

The logistic model was used early this century to examine population trends in the United States. Interest in this model was revived when the populations of England, Scotland and the United States were well-fitted by logistic curves[4], but a jump in the parameters occurred just after the Second World War. The same phenomenon was found for Australia and New Zealand[5] at the same time. Thus there are apparently logistic-type regimes which persist until some major event occurs.

The parameters of the best-fitting curves for the various models were computed with a nonlinear least-squares algorithm[6]. The logistic provided an excellent fit to the world and continental population data available at 5-year intervals from 1950 to 1985 (ref. 1). The predicted world population for 1992 is 5.48 billion, in agreement with the UN figure of April 1992.

If the present logistic regime persists, the world population will double in 47 years and the eventual world population will be 23.8 billion. This will be practically attained by the year 2200. The estimates of the final continental populations are as follows: Africa, 10.57 billion; Asia, 11.08 billion; Commonwealth of Independent States, 320 million; Europe, 556 million; Latin America, 899 million; North America, 336 million; and Oceania, 40 million. The figure shows the predicted population trajectories for the world and major continental regions. In all cases the predictions are much higher than those of the United Nations.



World and major continental population trajectories to the year 2250 obtained from the best-fitting logistics.

The malthusian and gompertzian models also gave good fits to the data to 1985. But the malthusian gives rise to an infinite population and the gompertzian predicts that saturation will occur in a few millennia at a final population of 1,062 billion, which seems unrealistic. (Note that the gompertzian is used mainly in biology to describe the development of masses of cells, as in certain organisms, their organs or tumours[7].)

The logistic has been derived as a model for populations which may disperse throughout a finite habitat[8]. It will be of interest to see when saturation effects are apparent in the world population, as they are already in the Commonwealth of Independent States, Europe and North America.

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