atom of size σ^2 at 0 yields the Gaussian component. An outstanding feature of the book is the list of unsolved problems and remarks at the end of most sections. This book should be of interest to the researcher and provides a valuable background for workers in the field.

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Characterizations of the Normal Probability Law.

A.M. Mathai and G. Pederzoli. New York: Halsted Press, 1978. vii + 149 pp. $8.50.

This book deals mainly with various mathematical characterizations of the univariate normal distribution, although the last two chapters consider similar characterizations of multivariate normal laws and information measures.

In general, characterization theorems are derived in the following manner. One first finds a property of the normal distribution and uses this to derive a functional or differential equation. Additional assumptions are then made as necessary to insure that there is a unique solution to these equations and that the unique solution is the normal distribution. For example, the following is shown to characterize the normal distribution. Let $X_1, \ldots, X_n$ be independent random variables. Then $\sum_{i=1}^n a_i x_i$ and $\sum_{i=1}^n b_i X_i$ are independent, where $a_i b_i \neq 0$ if and only if X_i is normally distributed. The normal distribution is also characterized through independence of linear and quadratic forms, and through the property of linear regression coupled with properties of conditional variance. Functional equations and differential equations are also presented which yield the normal law.

Characterization theorems are of interest for several reasons, primarily because they allow us to assess the exact nature of the assumptions made when we assume a particular model. In this light, this book should be useful because the normal model is the most common statistical model. The topic of the book is also useful for deriving statistical tests of normality. A chapter is presented outlining applications of characterization theorems to tests of hypotheses. Exercises are included if one wants to use this book as a basis of a course on characterization theorems.

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Applications of Mathematics, Volume 7: Game Theory.

N.N. Vorob'ev. New York: Springer-Verlag, Inc., 1977. xi + 178 pp. $16.80.

This book provides an excellent, rigorous introduction to the mathematical structure of game theory. The author, in the preface, states that the book requires only elementary tools of linear algebra and calculus, apart from one use of Brouwer's fixed point theorem; while a translator's remark claims that the book is suitable for a senior level undergraduate course in mathematics in which the book, plus some additional material, is covered. Both remarks are fully accurate and give a good indication of the level of difficulty. The exposition is extremely clear throughout and proofs are fully and patiently done.

To give an idea of the book's organization, the first chapter (pp. 1–55) begins with a five-page general description of an n-person variable-sum game which provides a framework for virtually the whole of the book. The remainder of the chapter is a thorough treatment of finite two-person zero-sum games with various examples interspersed along the way. The second chapter (pp. 56–89) is devoted to two-person zero-sum infinite games; thus two-person zero-sum games occupy just over half the text.

The third chapter (pp. 90–115) takes up noncooperative games. The first third of it is an exposition of Nash's theorem on the existence of an equilibrium point for n-person variable-sum finite games, including Nash's beautiful proof, and the remainder of the chapter takes up bi-matrix games and various special cases and examples. The final chapter (pp. 116–163) deals with cooperative games. At the outset of Chapter 4, the characteristic function (which assigns a scalar value to each coalition) is derived from the normal form game which is set out at the start of the book and used in the first three chapters. Many basic properties and concepts are introduced and, where appropriate, proved. The core, the von Neumann–Morgenstern solution, and the Shapley value are the equilibrium concepts covered and, as in earlier chapters, various examples are intermixed. None of Nash's work on cooperative games is covered. Exercises, a brief bibliography, and an index occupy the remainder of the volume.

For a book designed to serve mathematicians, economists, and others, I would have preferred some minor alterations in the selection of material. Most of the examples and simple games are, in my view, of dubious value. Had they been omitted, the space released could have contained an exposition of Nash's bargaining problem and, in Chapter 3, a generalization of Nash's theorem on noncooperative equilibrium could have been stated and proved for which strategy sets are compact and convex and mixed strategies are not used.

As a book for economists, it devotes too much space to zero-sum games, as well as to games requiring the possible use of mixed strategies. Economic situations are rarely zero-sum and, under most circumstances in which game theory is applied to economic phenomena, it is unreasonable to assume that decision makers use mixed strategies. It should also be noted that the book contains no applications of any consequence to economics or other disciplines. While there are a few purportedly economic examples, they have merit only as simple illustrative games and not as economics.

The author makes no attempt to cover anything outside of the mathematics of game theory proper, and even that is only at an introductory, though rigorous, level. He succeeds very well at his chosen task and I know of no other book which undertakes the task of this one. Some (e.g., Owen 1968 and Burger 1963) cover more ground but are generally much more difficult; others (e.g., Davis, 1970) are not mathematically difficult but are not rigorous; and yet others (e.g., Aumann and Shapley 1974, Friedman 1977, and Harsanyi 1977) go to great depth in some areas but give little or no coverage to some central topics.

Anyone teaching a course in game theory would probably want to supplement this book with other readings; however, for a lucid and complete coverage of some of the most basic material in the area, it is first-rate.

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Models of the Stochastic Activity of Neurones.

Arun V. Holden. New York: Springer-Verlag, Inc., 1976. vi + 368 pp. $12.80.

This is Volume 12 in Springer-Verlag's *Lecture Notes in Biomathematics* series, designed to report recent developments in rapidly growing research fields. The author has produced an extremely comprehensive survey of those parts of the area of mathematical neurobiology which employ the methods of applied probability, applied statistics, information theory, and ordinary and partial differential equations.

The subject matter is the stochastic element which is ubiquitous in the activity of neurons. Such cells are electrically excitable and many have the capability of transmitting impulses (action potentials) which may alter the electrical states of other cells. There are billions of nerve cells in mammalian brain, spinal cord, and peripheral nervous systems, and a given cell may receive thousands of inputs from other cells. The sequence of impulses generated by a cell can be regarded as a random time series of all-or-none events.

Chapter 1 deals with random fluctuations in the potential across the nerve cell membrane. The classic phenomena of thermal noise and shot noise are briefly considered. There follows a section on "$1/f$" noise (a process whose spectral density is approximately an

inverse function of frequency for small frequencies). Models which predict this spectrum are studied in order to understand the microstructure of the channels through which ions cross the membrane. Chapter 2 introduces several models for the conversion of a stochastic input to sensory neurons (e.g., retinal cells receiving light stimuli) into a train of action potentials.

Chapters 3 and 4 discuss the (deterministic) Hodgkin-Huxley model for impulse generation together with some simplified versions which make the study of random fluctuations in threshold and ion conductances more tractable. There is an interesting application of stochastic phase plane analysis.

Chapters 5 through 11 probably have the greatest appeal to applied probabilists and statisticians. The standard characterizations of renewal processes, which occupy a substantial part of Chapter 5, can be found in many texts on stochastic processes but there is an advantage in having them summarized before proceeding with the applications.

The topics of Chapter 6 are random walk models, birth and death processes, and the main problem of first passage times, with and without reflecting barriers. The simplified models themselves generate quite difficult analytic problems. The next chapter concerns diffusion approximations for nerve membrane potential. One of these is the Wiener process for which the first passage time density is known for linear barriers. Introducing a fragment of physiological reality—the exponential decay of the potential between inputs—leads to the classic Ornstein-Uhlenbeck model for Brownian motion, for which the first passage time problem is much more difficult. The approach to the diffusions is through their Kolmogorov equations. There has been very little progress in finding first passage times to moving barriers.

Problems connected with the superposition of point processes are described in Chapter 8, and Chapter 9 addresses the interesting problem of the interaction of spike trains that have been generated at different parts of the same neuron. More complicated models for impulse generation with random inputs are discussed in Chapter 10, which includes waiting time problems for renewal processes and exit time problems for discontinuous Markov processes. In Chapter 11 the statistical properties of sequences of spontaneously occurring miniature potentials at neuromuscular junctions are investigated. One problem, the adequacy of the Poisson approximation to the sequences, was one of the first problems in neurobiology to require a statistical analysis; see, for example, Cox and Lewis's *The Statistical Analysis of Series of Events*. Chapter 12 contains a survey of models for the activity of ensembles of interacting neurons where some progress has been made, but the author emphasizes the preliminary nature of the modeling. The last chapter introduces Shannon's ideas on information theory, and applications to single neurons and chains of neurons are briefly discussed.

This book would be a useful reference for a course in applied stochastic processes. Some knowledge of neurophysiology and neuroanatomy is desirable to make it completely understandable. I found this volume an extremely valuable source of references in theoretical neurobiology and came across many new and interesting problems. A possible criticism is that the author has tried to cover too much, so that various results are uncritically transcribed and continuity sometimes suffers. In summary, applied probabilists and statisticians will find a multitude of fascinating problems in this generally well-written survey of a field in which workers can do truly pioneering research.

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Clinical Biostatistics.

Alvan R. Feinstein. St. Louis: C.V. Mosby Company, 1977. xii + 468 pp. $13.50 (paperback).

This collection of 29 essays is taken verbatim from the 40 or so that have appeared in *Clinical Pharmacology and Therapeutics*. The book is aimed at those involved with clinical research, both physicians and consultant statisticians. Some familiarity with data analysis techniques is assumed, as is some knowledge of clinical medicine. The topics covered range widely, and include drug surveillance, case control and death certificate studies, surrogate measures of disease, "hard" versus "soft" data, and sampling methods. Dr. Feinstein is interested in how a clinical researcher transforms an anonymous citizen into a statistical table entry in a published report. The transformation is described as a harrowing one, and each pitfall along the way is pointed out with uncommon zest. In fact, for those unfamiliar with the admittedly informal style of the essays, the barrage of coined words, unexpected phrases, rhymes, and almost puns may either delight or distract. A glossary would be helpful.

The chapters on the architecture of clinical research outline the medical (or biological) considerations in research design, in contrast to the usual statistical issues in the design of experiments. Cautions on the use of case control studies, difficulties with cross-sectional studies, and problems in the use of established records are considered in other chapters. Sometimes these are illustrated with examples of the proposed errors.

The chapter called "A Primer of Multivariate Analysis" provides a sort of consumer's eye view for some of these procedures, and thus should be of interest to the consultant who proposes their use to a clinical investigator.

The final chapters are on stratification in the analysis of clinical trial data. The proposed techniques are supposed to provide more accessible and interpretable results than the multivariate techniques.

These essays raise a number of provocative points on the interaction between clinician and statistician. Statisticians who consult on the design or analysis of clinical studies should find the book interesting and relevant, whether or not they agree with all of its conclusions.

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Research in Labor Economics: An Annual Compilation of Research. Volume I.

Ronald G. Ehrenberg (ed.). Greenwich, Conn: JAI Press, 1977. ix + 376 pp. $17.50.

Research in Labor Economics consists of a collection of what would otherwise appear in the literature as journal articles except that the authors present more specific descriptions of the research to help the reader understand what the author did. The content of this first volume (1977) is varied. Included are review articles (Sherwin Rosen on human capital, Richard Freeman on manpower requirements, Donald Parsons on turnover); new theoretical approaches (John Pencavel on work effort); policy-related articles (Frank Brechling on experience rating of unemployment insurance); articles on macroeconomics (Toikka, Scanlon, and Holt on the labor market, and Burton and Addison on inflation); and an article based on a simulation model (Ralph Smith on participation and unemployment).

In a sense the volume really amounts to a journal except that it appears annually rather than quarterly and hence a full year's subscription can be bound in advance. The strategy of the editor of this journal appears to be unique: he would rather publish a few complete articles than numerous brief articles, and he does most of the reviewing himself, thus avoiding problems of negligent referees who take excessive amounts of time to provide a review. Judging from Volume I, the editor has high standards, and all of these papers could probably have found places in respectable journals.

Because the subjects covered do not have a common, unifying theme, a more specific discussion of the volume necessarily involves commenting on the individual articles. Those which I found most interesting include the review article by Rosen and the unemployment insurance piece by Brechling.

Sherwin Rosen's survey of human capital models and empirical evidence can be viewed as an updating of the 1970 piece by Jacob Mincer on the income distribution (*Journal of Economic Literature*, 1970). The human capital literature is enormous, yet the evolution of the theoretical models and additional empirical evidence and puzzles is traced out in a fashion comprehensible to the person who is not deeply familiar with the area while maintaining the interest of the connoisseur. Empirical topics covered include the attempts to control for ability factors in analyzing returns to investments. In this research the problem of underidentification has abounded. Recent efforts suggest that future research will make greater use of econometric models proposed by Jöreskog and Goldberger. In terms of recent theoretical work on integrating labor-leisure models with capital accumulation, Rosen's article has a good, brief discussion of this topic and concludes that empirical work on the full model may be feasible (p. 33).