

THE METALANGUAGE OF NEURON GROUPS

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Abstract

This paper deals with a language (a metalanguage) describing the functioning of neuron collections based on the concept of transitivity of impulse train stochastic dependence. Potentialities of the language for ascertaining the information coding principles of the nervous system are demonstrated.

1. Introduction

Over the past few decades both theorists and experimentalists have suggested at least ten qualitative theories (partly phenomenological, partly hypothetical) which give plausible explanations of brain functioning based on the idea of interacting neuron ensembles (cartels, functionally equivalent units, clusters, groups, etc.) (Hebb, 1949; Chang, 1950; Marr, 1969; Freeman, 1972; MacGregor and Radcliffe, 1973; Brindley, 1974; Perkel *et al.*, 1975; Somjen, 1975; Dunin-Barkovskij, 1978; Kohonen, 1978; Grinvald *et al.*, 1981; Sokolov, 1981; Mountcastle and Edelman, 1981; Kovbasa *et al.*, 1984a; etc.). Far from trying to belittle their conceptual merits, we, nevertheless, would like to point to a common disadvantage of these theories: the ambiguity of qualitative description and, if the mathematical models are employed, a number of assumptions which cannot be corroborated experimentally, at least at the present time.

This paper aims at establishing elements of a quantitative information theory of neuron systems all the theses of which can be verified by the experimental means available to the researchers.

It seems that dependence as a phenomenon is common to all material relations (independence is a specific case of dependence). Historically, it is customary to distinguish between three main types of dependence (if the latter is considered from the quantitative point of view). These are determinate (functional) dependence, correlation dependence and the most general of all — stochastic dependence. In

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studying the nervous system neurobiologists and neurocyberneticians focus great attention on gradual and peak potentials of neurons which, from the mathematical standpoint, are realizations of continuous random processes with a continuity of states. The analysis of these realizations has demonstrated that the electrical activity processes of neurons can be strictly determinate (pacemakers) or random: stationary in the broad sense (generally the background activity) or non-stationary (generally the induced activity). Therefore, when trying to establish the principles of signal coding by the nervous system it is quite natural to evaluate the dependence of these processes using all three foregoing types. In most general cases one can always ascertain the presence or absence of the stochastic dependence. It is precisely this operation that can be performed in any conditions, for which reason this paper treats the stochastic dependence (SD) between the neuron trains as the "invariant" characteristic of neuron system functioning (Grenander, 1978).

2. The Principle of "Ring" Stochastic Dependence

Let us evaluate the SD of impulse trains using a statistical method proposed by the author (Kovbasa, 1980), though this choice is not necessary for the further presentation. To apply the method in practice we assume we have synchronous records of neurons in question. Modern optical techniques of action impulse recording make it possible to realize simultaneous output from 100 and more channels (Grinvald *et al.*, 1981). Introduce now the concept of a group. Hereinafter, we shall consider the group to be a collection of neurons integrated by the object relation, the meaning of which will be clarified later. Call the number of neurons $n = 1, 2, 3, \dots$ in a group "the power". Assume that by applying the aforementioned statistical method we find out whether the impulse train from the i -th neuron depends stochastically on that from the j -th neuron of the group, $i \neq j, i, j = 1, 2, \dots, n$. For large n this operation can be performed only with the aid of a computer. Present the result as a matrix calling it the matrix of stochastic dependence A . If the trains from the i -th and j -th neurons are stochastically dependent (for the adopted level of significance — see Kovbasa, 1980), assume the element at the intersection of the i -th row and j -th column of this matrix, $i \neq j, i, j = 1, 2, \dots, n$, to be equal to unity. Otherwise, let it be equal to zero.

Consider the properties of the matrix A . First, it consists only of zeroes and units, it is square and symmetric. In the graph theory such matrix is usually called the incidence matrix. Second, its elements on the main diagonal are always units, since they express the stochastic dependence of the i -th train on itself, $i = \overline{1, n}$. Third, it depends on the group power n , the interval of recording T and the level of significance α . Call the condition of a group of neurons described by a particular SD matrix the "stochastic condition" (SC). We proceed here from the assumption that before using the interdependence estimation method the train from each neuron of the group is checked first to make sure that it is really the train from one i -th neuron, $i = \overline{1, n}$, and second, to verify whether a significant error is introduced into the SD matrix due to interference of the action potential of summary electroneuronograms. This can be

achieved by using quantitative techniques developed earlier by the author *et al.* (Nozdrachev *et al.*, 1982; Kovbasa *et al.*, 1983, 1984b).

Assume that an event took place residing in the fact that the SD exists between the trains from the i -th and k -th neurons, $i \neq k; i, k = \overline{1, n}$. Assume further that the SD also exists between the k -th and j -th neurons; in this case it certainly exists between the i -th and j -th neurons, $i \neq k \neq j; i, k, j = \overline{1, n}$, provided that

$$(1) \quad H(i, j) \leq H(i|k) + H(j|k),$$

where $H(i|k)$ and $H(j|k)$ are conditional entropies of trains i and j in respect of the train k , and $H(i, j)$ is their common entropy. The opposite is obviously not true. Let us postulate the correctness of the transitivity principle as applied to the SD, i.e. the correctness of the "ring" SD concept. Relationship (1) is to be considered as a condition in which a collection of individual neurons acquires the object quality thus becoming an association. Let us demonstrate the sufficiency of this condition to confirm the transitivity of the SD between the trains:

(a) Assume that impulse interval trains from three neurons can be considered as realizations of random values X_1, X_2, X_3 . Assume that X_2 is dependent on X_1 and X_3 and vice versa. Then the following relationships are valid: $H(X_1|X_2) < H(X_1)$ and $H(X_3|X_2) < H(X_3)$, where $H(\cdot)$ stands for entropies of argument (\cdot). Take the sum of the left-hand and right-hand sides of these inequalities. Then we have

$$(2) \quad H(X_1|X_2) + H(X_3|X_2) < H(X_1) + H(X_3)$$

Assume that condition (1) is complied with. Then, obviously, we have $H(X_1, X_3) \leq H(X_1|X_2) + H(X_3|X_2) < H(X_1) + H(X_3)$, i.e. $H(X_1, X_3) < H(X_1) + H(X_3)$, which means that values X_1 and X_3 are interdependent. Thus the sufficiency is proved.

(b) Find out what meaning can be attached to relationship (1) by considering an example of steady trains.

Inasmuch as X_1, X_2, X_3 are continuous random variables, we can explicitly write relationship (1) as

$$(3) \quad - \int_0^{\infty} \int_0^{\infty} f_{13}(x_1, x_3) \ln f_{13}(x_1, x_3) dx_1 dx_3 \leq \\ \leq - \int_0^{\infty} \int_0^{\infty} f_{12}(x_1, x_2) \ln f_{12}(x_1|x_2) dx_1 dx_2 - \int_0^{\infty} \int_0^{\infty} f_{32}(x_3, x_2) \ln f_{32}(x_3|x_2) dx_3 dx_2$$

For definiteness consider the case when X_1, X_2, X_3 are described by the following differential functions:

$$(4) \quad f_i(x_i) = \frac{1}{ek} \frac{\lambda_i \exp(-\lambda_i x_i)}{1 + \lambda_i x_i}, \quad \lambda_i > 0, x_i \geq 0, \\ k = \int_1^{\infty} x_i^{-1} \exp(-x_i) dx_i.$$

It is easy to check that

$$\int_0^{\infty} f_i(x_i) dx_i = 1$$

and

$$\int_0^{\infty} f_{ij}(x_i, x_j) dx_j = f_i(x_i),$$

where

$$(5) \quad f_{ij}(x_i, x_j) = \frac{1}{ek} \lambda_i \lambda_j \exp(-\lambda_i x_i - \lambda_j x_j - \lambda_i \lambda_j x_i x_j); \quad i \neq j; \quad i, j = \overline{1, 3}.$$

It is seen from these expressions that

$$f_{ij}(x_i, x_j) \neq f_i(x_i) f_j(x_j), \quad i \neq j.$$

Let us find what meaning can be attached to inequality (1) by the use of distribution parameters (4). To this end substitute expressions (4) and (5) into inequality (3), having recalled that

$$f_{ij}(x_i | x_j) = \frac{f_{ij}(x_i, x_j)}{f_j(x_j)}$$

As a result of some awkward transformations we have the following relationship equivalent to inequality (3):

$$(6) \quad \ln \lambda_1 \lambda_3 \leq \frac{2ek_1 + \ln k + e - 1}{1 - ek},$$

where

$$k_1 = \int_1^{\infty} x_i^{-1} \exp(-x_i) \ln x_i dx_i.$$

With regard to specific values of $k_1 = 0.09783$ and $k = 0.21942$ (the accuracy of the calculation is 10^{-5}), we finally obtain $\ln \lambda_1 \lambda_3 \geq 1.51678$.

We now discuss the result. First, in the domain of single-parameter functions (4) the effect of X_2 on relationship (6) is realized through the mediation of coefficients k_1 and k_2 . Second, from (6) it is seen that to comply with the transitivity of the SD, the product of intensities of steady trains X_1 and X_3 (equal to $ek\lambda_1/(1-ek) = 1.46917\lambda_1$ and $ek\lambda_3/(1-ek) = 1.46917\lambda_3$, respectively) shall not be too small. The exact lower bound is determined by the right-hand side of expression (6).

As is well known, the presence of bonds between elements (in our case the

availability of the SD) is the necessary but not sufficient condition for gaining the object quality by a collection of elements. Let us clarify it graphically. Assume that the presence of the SD between the i -th and j -th neurons, $i \neq j; i, j = \overline{1, n}$ (in Fig. 1a they are designated by circles), is shown by a connection line, and its absence — by the absence of the latter. Then each SD matrix corresponds to one, and only one, connection pattern and vice versa. It is clear that this pattern is nothing but a completely non-oriented graph multiple to 1. The "ring" SD principle effects the situation in which each neuron group of power n has finite and strictly definite number of different SCs. In Fig. 1 these SCs are shown by five graphs for the group of power $n = 4$. Near the corresponding graph the SD matrix is presented. Hereinafter $S_n^{(k)}$ stands for the k -th SC of the neuron group of power n . Indeed, the situation shown by the graph in Fig. 1b cannot occur, because, according to the ring SD principle, the presence of the SD between neurons No. 1 and No. 2 and between neurons No. 2 and No. 3 necessitates the SD between neurons No. 3 and No. 4. For the same reason any SC in which a neuron group of power n finds itself by the moment $t = T$ can be, by a simple renumbering of the neurons, reduced to a form where its SD matrix (1) consists only of units or (2) consists of square submatrices composed only of units and those composed only of zeroes. This is depicted in Fig. 1a. In terms of the graph theory it means that each SC is characterized either by a complete graph or by a zero graph, or can be presented by a union of complete graphs (in Fig. 1a, these are $S_4^{(5)}, S_4^{(1)}, S_4^{(2)}, S_4^{(3)}, S_4^{(4)}$). Hence, the ring SD principle can be also laid down as follows: a group of neurons can change from any allowed SC only to the SC described by a union of complete graphs. The number of different allowed SCs, S_n , for a group of power n can be defined by the formula

$$(7) \quad S_n = \begin{cases} 1 + \sum_{j=2}^n \left[\frac{n}{j} \right] + \sum_{j=2}^{n/2} \sum_{l=j+1}^{n-j} \sum_{p=1}^{[n/l]} \left[\frac{n-lp}{j} \right], & \text{if } n \text{ is even} \\ 1 + \sum_{j=2}^n \left[\frac{n}{j} \right] + \sum_{j=2}^{[n/2]+1} \sum_{l=j+1}^{n-j} \sum_{p=1}^{[n/l]} \left[\frac{n-lp}{j} \right], & \text{if } n \text{ is odd.} \end{cases}$$

Here $[n/j]$ is the integral part of the number n/j ; if $(n-lp) < 0$, then $\left[\frac{n-lp}{j} \right] = 0$.

From this formula it transpires that the following recurrent relationships can be obtained for S_n :

$$S_n = 2S_{n-1} - S_{n-2}, \quad \text{if } 12 \geq n \geq 3 \text{ is odd}$$

$$S_n = 2S_{n-1} - S_{n-2} + \frac{n-2}{2}, \quad \text{if } 12 \geq n \geq 4 \text{ is even.}$$

Specifically, Conclusion 1 (see next section) specifying the number of different SCs depending on n holds true for groups of power $n = 1, 2, \dots, 9$.

3. The Phenomenon and Effect of Stochastic Entrainment

Inasmuch as the SC of a neuron group is determined based on impulse activity records during certain time T , the change from one SC to another is registered by the variation of the SD matrix at "sliding" of neurograms with the time slot of duration T . It will be quite natural, considering the type of the SD matrix, to introduce for the group of power n the following quantitative characteristics of the change from one allowed SC to another: $\xi^{(n)} = \frac{1}{2}(X^{(n)}, Y^{(n)})$, where $X^{(n)}, Y^{(n)}$ are two components of the random value $\xi^{(n)}$. The first component gives the number of zero elements of the SD matrix for the initial SC, the second one — for the realized SC. The coefficient is equal to $\frac{1}{2}$, since the matrix A is symmetric. Each of the components takes on the values of nonnegative integers not exceeding $n(n-1)$. However, due to the existence of the ring SD, not all integers ranging from 0 to $n(n-1)$ can be the values of the said components. For instance, for the group of power $n=4$ (see Fig. 1a) the value $X^{(n)} = x^{(n)} = 4$ is ruled out. The total number of possible values assumed by each of the components $X^{(n)}$ and $Y^{(n)}$ depends on the power of the group, n , and equals exactly S_n . The case $X^{(n)} = Y^{(n)}$ corresponds to the situation when the SD matrix of the initial SC is equal to that of the realized SC.

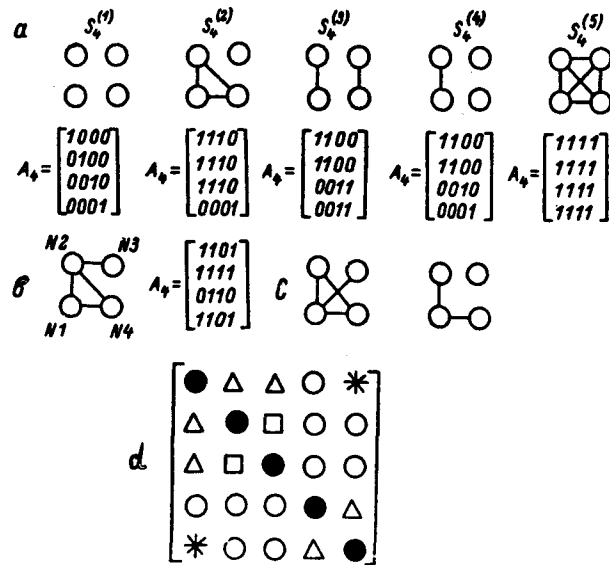


Fig. 1 Stochastic conditions (SC) are shown by five graphs for the group of power $n=4$. Near the corresponding graph the SD matrix is presented. (a) — SC are shown by a union of complete graphs (b), (c) — SCs which cannot occur according to the ring SD principle (d) — The structure of the matrix of change $\|P_{ij}\|$, depending on the number of "formations" and "breaks" of SD.

Let us introduce now another quantitative characteristic of a neuron group dependent on the power n :

$$r_n = S_n - n.$$

Call it the *cleavage number*. As is clearly seen from formula (7) $r_n \geq 0$. Consider the meaning of the value r_n . It indicates how many SCs comprising equal number of SD rings are there in the neuron group. In terms of the graph theory the same can be formulated as follows: the number r_n indicates how many SCs, described by the graphs representable by unions of the same number of complete graphs, are there in the neuron group of power n . This is illustrated by Fig. 1a depicting the graphs corresponding to 5 SCs of the neuron group of power $n=4$. In this example $r_4 = S_4 - 4 = 5 - 4 = 1$. It means that one of the allowed SCs has a "twin" in the sense that it is representable by a graph containing the same number of SD rings (complete graphs). Indeed, as is seen from Fig. 1a, this is true for the SCs $S_4^{(2)}$ and $S_4^{(3)}$: each of them is representable by a graph containing two SD rings (i.e. both SCs are representable by the union of two complete graphs).

Conclusion 1									
The number of different allowed SCs depending on n									
n	1	2	3	4	5	6	7	8	9
S_n	1	2	3	5	7	11	15	22	29

Thus, each neuron group is characterized quantitatively by the following three values: n — power of the group, S_n — number of allowed SCs, r_n — cleavage number.

Consider two neuron groups of power n_1 and n_2 , respectively. Assume that by a certain moment $t = T$ both groups are described by complete graphs. Assume that by the moment $t = T_1 > t = T$, by virtue of the neuronogram processing, it is found that (a) each of the neuron groups is described by a complete graph and (b) there is a SD between a neuron belonging to the group of power n_1 and that belonging to the group of power n_2 , and vice versa. Therefore, at the moment $t = T_1$ the SC of the two groups under consideration will be described by one complete graph. Call this phenomenon the "phenomenon of stochastic entrainment". On the other hand, assume that in the neuron group of power n_1 (where all neurons are believed to be stochastically dependent on each other by the moment $t = T$) one of the neurons has sharply changed its impulse generation pattern by the moment $t = T_1 > t = T$ and ceased to be stochastically dependent on other neurons of the group. Thereby it "left" the group in question and probably was entrained by the neurons of another group which formed the SD ring of their own. The neurons of the initial group of power n_1 form the SD ring as before, but this time with the "aid" of $n_1 - 1$ neurons only. Further on let us call the SD ring a ring of the order $k = 1, 2, \dots$, if it is formed by the SD of k neurons.

As has been demonstrated above each group of neurons of power n is characterized by a strictly definite number of allowed SCs, some of these SCs being described by the same number of SD rings (which takes place when $r_n > 0$). Name these last-mentioned SCs the “degenerated” ones. Conclusion 2 lists the values of the cleavage number for the groups of power $n = 1, 2, \dots, 9$. The degenerated SCs, though containing the equal number of SD rings, are characterized by the rings of different order k . The situation for the power group $n = 4$ is illustrated in Fig. 1a.

Conclusion 2									
Values of the cleavage number r_n depending on n									
n	1	2	3	4	5	6	7	8	9
r_n	0	0	0	1	2	5	8	14	20

The phenomenon of stochastic entrainment is accompanied by the so-called entrainment effect. To explain this effect, again consider two neuron groups of powers n_1 and n_2 , respectively. Assume that at the moment $t = T$ each of these groups is described by the SC characterized by just one SD ring (of the order $k = n_1$ and $k = n_2$, respectively). To put it another way, in each group all neurons are stochastically dependent on each other, whereas any two neurons from different groups are stochastically independent. Let the group of power n_1 have S_{n_1} of different SCs, and that of power $n_2 - S_{n_2}$ of the same. Assume that by the moment $t = T_1 > t = T$ the entrainment took place between these two groups resulting in the formation of a group of power $n_1 + n_2$ described by one SD ring of the order $k = n_1 + n_2$. Denote the number of different allowed SCs of the new group by $S_{n_1+n_2}$. It turns out that $S_{n_1+n_2} > S_{n_1} + S_{n_2}$ for small n and $S_{n_1+n_2} \gg S_{n_1} + S_{n_2}$ for large n , which is inferred directly from the recurrent relationships for S_n . It is this violation of the additivity of the SC number which furnishes the entrainment effect.

4. Principles of Coding the Signal by SD Rings

Based upon the above arguments, it can be stated that a group of neurons functions so that it can change from one allowed SC to another, or remain in some fixed SC. The change is possible at any moment of time (formally speaking, the retention of the initial SC may be regarded as a change). There are no “irreversible” SCs. Assume that a group of power n has $S_n = m$ different allowed SCs. Each of these SCs, $S_n^{(1)}, S_n^{(2)}, S_n^{(3)}, \dots, S_n^{(m)}$ can be considered as an event. Obviously, these events form a complete group. Consider the matrix of change

$$P_m = \|p_{ij}\|, \sum_{j=1}^m p_{ij} = 1; i = \overline{1, m},$$

where p_{ij} is the probability of the neuron group change from the SC $S_n^{(i)}$ to the SC $S_n^{(j)}$, $i, j = \overline{1, m}$, by the moment $t = T_1$. Ascertain what will be the appearance of this matrix depending on the number of d — the number of “formations” or “breaks” of the SD between neurons of the groups. The existence of the ring SD results in the fact that the “formation” or “break” of a certain number of SDs between the group neurons by the time $t = T_1$ (in the SC graph it is reflected by the presence or absence of a certain number of branches) is conditioned not only by the power of the group, n , but also by the SC of the entire group at the initial time moment. To make this clear, assume that by a certain time moment $t = T$ a neuron group of power $n = 4$ finds itself in the SC with two SD rings of the order $k_1 = 3$ and $k_2 = 1$. This SC $S_4^{(2)}$ is already familiar to us (see above). Being in this SC, the group cannot “form” or “break” exactly one SD (in the graph it was depicted by a branch). This is inhibited by the ring SD principle. Indeed, the neuron group in $S_4^{(2)}$ cannot change to the SCs represented by the corresponding graphs in Fig. 1c. For the same reason, having retained the SC $S_4^{(2)}$, the group cannot “form” two SDs, however it can “break” two SDs.

It should be noted that the above discussion is valid for a group of fixed power n . Thus, a neuron group of fixed power n can always change from one allowed SC $S_n^{(i)}$ to another allowed SC $S_n^{(j)}$, $i, j = 1, 2, \dots, m$. However, being in some allowed SC $S_n^{(k)}$, $k = \overline{1, m}$, the group cannot “form” or “break” an arbitrary number of SDs between the neurons. In particular, for a group of power $n = 4$ ($m = 5$ — five allowed SCs: $S_4^{(1)}, S_4^{(2)}, S_4^{(3)}, S_4^{(4)}, S_4^{(5)}$) we have, depending on d , the matrix of change shown in Fig. 1d. In this figure non-zero probabilities p_{ij} at $d = 0$ are denoted by shaded circles; non-zero probabilities at $d = 1$ — by ordinary circles; non-zero probabilities at $d = 2$, if there occurred two “formations” or two “breaks” — by triangles; non-zero probabilities at $d = 2$, if there occurred a “formation” and “break” or “break” and “formation” — by squares; and non-zero probabilities at $d = 3$ — by stars. Fig. 2 depicts the graphs describing changes of SCs depending on n — the power of the group, and d — the number of “formations” and “breaks” for the case under consideration. Large circles stand for the allowed SCs.

It is easily comprehended that the “formation” of the SD is nothing but the “entrainment” of a neuron by the SD ring, whereas the “break” is the escape of a neuron from the SD ring. Consequently, each group of neurons can code the signal by forming (or breaking) the ring SD. Indeed, a group of neurons codes the signal, if its SC changes, which becomes clear after estimating the interdependence of impulse trains for the adopted level of significance α .

It is well known that any information is a sequence of elementary messages or a sequence of random variables $\xi_1, \xi_2, \xi_3, \dots$. Each elementary message can be considered as a discrete random variable assuming one of b values ($b = 1, 2, \dots, B$) with the probability $P(\xi)$. According to the information theory each individual message, i.e. each realization of a random variable $\xi = i$, corresponds to its “word” $V(i) = (\eta_1^i, \eta_2^i, \dots, \eta_{l_1}^i)$ in a certain alphabet Al (l_1 is the length of the word). The complete collection of these words (their number equals B) forms the code. Having known the code, upon realization of the message $\xi_1, \xi_2, \xi_3, \dots$, this message is written in terms of the alphabet Al . It takes the form $V(\xi_1)V(\xi_2)\dots = \eta_1\eta_2\dots$, where $V(\xi_i)$ is the sequence of letters constituting the alphabet Al .

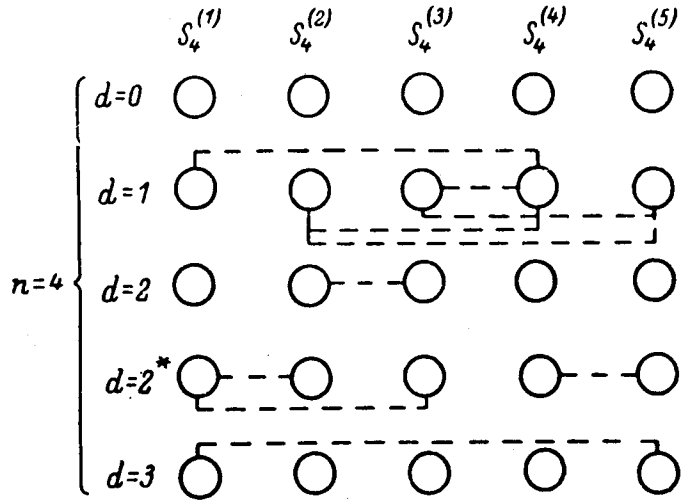


Fig. 2 Graphs describing changes of SCs depending on d – the number of “formations” and “breaks” of SD (for $n=4$).

By considering a neuron group of power $n = 4$ let us demonstrate, proceeding from the phenomenon of stochastic entrainment, how such a message is formed and what the code will be. All messages which can be coded by this group are related to five discrete two-dimensional random variables $\xi_1^{(4)}, \xi_2^{(4)}, \xi_3^{(4)}, \xi_4^{(4)}, \xi_5^{(4)}$ each of whose value characterizes the change from one SC to another. To put it another way, these random variables are nothing but two-dimensional random variables whose component values represent the number of zero elements of the SD matrix (it would be remembered that we first introduced these variables in section II of this paper). Indeed, $\xi_1^{(4)}$ is described by the following law:

$\frac{1}{2}(x_i, x_j)$	6;6	3;3	4;4	5;5	6;6
P_{ij}	P_{11}	P_{22}	P_{33}	P_{44}	P_{55}

Possible values of components shown in the first line and their non-zero probabilities (the second line) correspond to the case with $d=0$, when not a single SD of the group undergoes changes (i.e. neither “forms” nor “breaks”). $\xi_2^{(4)}$ is described by the following law:

$\frac{1}{2}(x_i, x_j)$	6;5	5;6	3;0	0;3	4;0	0;4	5;3	3;5	5;4	4;5
P_{ij}	P_{14}	P_{41}	P_{25}	P_{52}	P_{35}	P_{53}	P_{42}	P_{24}	P_{43}	P_{34}

It corresponds to the case with $d=1$, when one SD of the group changes; $\xi_3^{(4)}$ is determined as

$\frac{1}{2}(x_i, x_j)$	6;3	3;6	6;4	4;6	4;0	0;4
P_{ij}	P_{12}	P_{21}	P_{13}	P_{31}	P_{45}	P_{54}

which corresponds to the case when two SDs “form” or “break”, i.e. $d=2$ (exactly two SDs); for $\xi_4^{(4)}$ we have

$\frac{1}{2}(x_i, x_j)$	2;4	4;2
P_{ij}	P_{23}	P_{32}

This is the case when one SD “forms” and another “breaks” or vice versa; and finally for $\xi_5^{(4)}$ we have

$\frac{1}{2}(x_i, x_j)$	0;5	5;0
P_{ij}	P_{51}	P_{15}

which corresponds to the case when 3 SD undergo changes. Obviously, the following equalities are complied with:

$$\begin{aligned}
 P_{11} + P_{22} + P_{33} + P_{44} + P_{55} &= 1; \\
 P_{14} + P_{41} + P_{24} + P_{42} + P_{25} + P_{52} + P_{34} + P_{43} + P_{35} + P_{53} &= 1; \\
 P_{12} + P_{21} + P_{13} + P_{31} + P_{45} + P_{54} &= 1; \\
 P_{23} + P_{32} &= 1; \\
 P_{15} + P_{51} &= 1,
 \end{aligned}$$

which is perfectly in line with the well-known property of the matrix of change

$$\sum_{i=1}^5 \sum_{j=1}^5 p_{ij} = 5.$$

It is quite natural to juxtapose the random variable $\xi_1^{(4)}$ with a word containing zero letters, i.e. with the divider of the words; the variable $\xi_2^{(4)}$ – with a word containing one letter; the variable $\xi_3^{(4)}$ – with a word containing two identical letters; $\xi_4^{(4)}$ – with a word containing two different letters; and the variable $\xi_5^{(4)}$ – with a word containing three identical letters. A group of 4 neurons cannot code any other words (with the different number of letters or having the different structure).

Apparently, the code corresponding to $\xi_1^{(4)}$ contains 5 words; the code corresponding to $\xi_2^{(4)}$ — 10 words; the code corresponding to $\xi_3^{(4)}$ — 6 words; the code corresponding to $\xi_4^{(4)}$ — 2 words; and finally the code corresponding to $\xi_5^{(4)}$ — 2 words. The total number of words is $5 + 10 + 6 + 2 + 2 = 25$. Out of this number only $25 - 5 = 20$ words are considered to be informative. The alphabet A consists of only two letters “F” and “B” which mean the “formation” or “break” of the SD, respectively.

5. The Metalanguage of Neuron Groups

Let us demonstrate the validity of the following statement.

Statement 1

A neuron group of power n has exactly n SCs which differ from one another by the number of SD rings. All other SCs come into being due to degeneration.

Proof

Consider a neuron group of power n . Let the SD structure of this group be described by a zero graph. Obviously, such SC consists of n SD rings each of order 1. Assume now that some two neurons form a SD ring of order 2, whereas other neurons of the group form SD rings of order 1. This time the SC is described by $n - 1$ SD rings. Next assume that some three neurons form a SD ring of order 3 and the rest of the neurons form SD rings of order 1. In this case the total number of SD rings will be $n - 2$. By continuing this operation we will have SCs containing $n - 3, n - 4, \dots, 3, 2, 1$ SD rings. Call these SCs the reference ones and designate them by $SC^{(n)}, SC^{(n-1)}, \dots, SC^{(3)}, SC^{(2)}, SC^{(1)}$. The reference SCs differ in the construction algorithm. In so far as the number of SD rings in a SC cannot exceed that of neurons in the group, any of the remaining $S_n - n$ SCs will contain SD rings equal in number to the quantity of SD rings in one of the aforementioned reference SCs. From this it follows that the total number of degenerated SCs in fact equals $r_n = S_n - n$. Statement 1 is proved.

As was mentioned earlier, each change from one SC to another is followed by the F or B of a certain number of SDs. Represent this fact by a column containing the corresponding number of symbols F and B , taking account of the dimension and composition of the column but not the succession of the symbols. Adopt the following classification of changes:

Call the change from one SC to another *the first-variety change*, if these SCs differ in the number of SD rings; *the second-variety change*, if they are described by the same number of SD rings; *the third-variety change*, if they are described by the same number of SD rings, but each ring of one SC can be juxtaposed with the same-order ring of another SC.

Let us now demonstrate the validity of the following statement.

Statement 2

Each first-variety change is defined by a column containing a different number of symbols F and B , whereas each second-variety change — by a column containing an equal number of the same.

Proof

Consider a first-variety change. Let there be a SC described by a number l_1 of SD rings; that is, the first SD ring has the order $K_1^{(1)}$, the second SD ring — $K_2^{(2)}, \dots$, the l_1 -th SD ring — $K_{l_1}^{(l_1)}$; so that $\sum_{i=1}^{l_1} K_i^{(1)} = n$, where n is the power of the neuron group. Using another terminology it can be said that there exist $l_1^{l_1}$ non-empty indistinguishable boxes containing $K_1^{(1)}, K_2^{(1)}, \dots, K_{l_1}^{(1)}$ indistinguishable items (say, of white colour), respectively. Let there be another SC described by the number of SD rings $l_2 < l_1$; at that, the first ring has the order $K_1^{(2)}$, the second — $K_2^{(2)}, \dots$, the l_2 -th — $K_{l_2}^{(2)}$; so that $\sum_{i=1}^{l_2} K_i^{(2)} = n$. By analogy, it can be stated that there exist $l_2^{l_2}$ non-empty boxes containing $K_1^{(2)}, K_2^{(2)}, \dots, K_{l_2}^{(2)}$ items (say, of black colour). The change from one SC to another will correspond to the operation of getting from the first composition of l_1 boxes the second composition of l_2 boxes by transferring each white item from box to box only once and preserving the same colour of items in the boxes. It is natural to assume that each box deprived of all its items will be withdrawn from the analysis. Let us study the operation in greater detail. Arrange mentally the boxes of both compositions against each other (not necessarily without omission) by first ordering them according to the number of items, i.e. so that the first box has the number of items not exceeding that of the second box, which, in turn, has the number of items not exceeding that of the third one, etc. Clearly, $l_1 - l_2$ boxes of the first composition will be left “without a pair”. Divide the operation into two stages: (1) equalize the number of boxes of both compositions, for which purpose get rid of the “unpaired” $l_1 - l_2$ boxes of the first composition by transferring all their items into the “pair” l_2 boxes; (2) equalize the compositions of the “pair” l_2 boxes (assume that this is feasible) by redistributing the items they contained before the first stage was started.

Consider now the first stage. Apparently, during this stage only F symbols will appear, their number being equal to that of the boxes liquidated. Indeed, using the previous terminology, each box is a SD ring which is described geometrically by a complete graph. So the adding of items into a box brings about the SD.

Consider now the second stage. During this stage the items are also transferred from one box into another, but this does not result in emptying of the box. According to the previous terminology it corresponds to the “break” and “formation” of the SD ring. In other words, it suggests the appearance of the symbol B and simultaneously — the symbol F . Hence, the first-variety change is always described by a column in which the number of F symbols differs from that of B symbols.

The second-variety change is in full conformity with the second stage of the operation and, therefore, is described by a column with an even number of lines containing equal number of F and B symbols. Statement 2 is thus proved.

Note 1

The liquidation of an "unpaired" box by transferring all its items into a "pair" one corresponds to the appearance of just one F symbol, because "inside" each box all items-neurons are organized in SD rings by the transitivity principle.

Note 2

It may well happen that the second stage will not complete the operation. Then as early as at the first stage, when liquidating the "unpaired" boxes, a part of items from individual (or probably from all) boxes is to be redistributed between different "pair" boxes. At that, the items taken last from each box to be liquidated will bring to the column only symbol F . And since for the first-variety change $l_1 > l_2$, the total balance of symbols will not be maintained in this case as well.

Note 3

The reverse first-variety change from the SC described by l_2 SD rings to the SC described by $l_1 > l_2$ SD rings corresponds to the construction of a column formed from the column realized at the direct change by substituting all B symbols for F symbols and vice versa. The direct and reverse second-variety changes are described by the same column.

We now demonstrate the validity of the following statement.

Statement 3

Among reference SCs of a neuron group of power n only three, viz. $SC^{(n)}$, $SC^{(n-1)}$, $SC^{(1)}$, are not capable of degeneration.

Proof

The number of degenerated SCs corresponding to a certain reference $SC^{(k)}$ (i.e. the SCs also corresponding k SD rings) can be determined by solving the problem on the number of distributions of n identical items between k non-empty indistinguishable boxes. This number is equal to the number of decompositions of a natural number n into k summands, such that $n_1 \leq n_2 \leq \dots \leq n_k$, $\sum_{i=1}^k n_i = n$. Obviously, the said decomposition is unique only for $SC^{(n)}$, $SC^{(n-1)}$ and $SC^{(1)}$. Statement 3 is proved.

A third-variety change can be considered as a special case of the second-variety change, since during this change the number of SD rings of the SCs under comparison remains constant. Call the effect resulting in this change — the effect of "rotation". It resides in the fact that in the process of functioning a neuron group, by changing its impulse generation pattern, redistributes its "significant" SDs (represented by the branches of the relevant graph) all the time remaining in the same SC. Quantitatively this effect is characterized by the rotation number indicating the number of operations by means of which the said SC can be realized with the aid of different "significant" SDs. For the SC described by l SD rings of the order k_1, k_2, \dots, k_l ; $\sum_{i=1}^l k_i = n$ this number can be obtained by computing the number of transfers by means of which n different items can be distributed between l non-empty indistinguishable boxes ($2 \leq l \leq n-1$), so that the first box would contain k_1 items, the second box — k_2, \dots , the l -th one — k_l items, $\sum_{i=1}^l k_i = n$. The limitation imposed on l is caused by the fact that two SCs containing 1 and n rings, respectively, are not capable of rotation. Specifically, the reference SCs capable of rotation have rotation numbers equal to C_n^l ($2 \leq l \leq n-1$), i.e. to the number of combinations of n elements taken l at a time.

We now introduce the following definition.

Definition

Call a column consisting of F and B symbols (alternatively, only of F or only of B) and appearing at the change of any variety, a word.

Thus, the words differ from each other in the number of column lines or, the number of lines being equal, in the proportion of symbols F and B entering into the column, but not in the succession of these symbols. These are the words which form the metalanguage.

The foregoing definition and three statements proved earlier make it possible to formulate *Statement 4*.

Statement 4

(1) Various first-variety changes of a group of fixed power n result in the appearance of words represented by a column consisting of i lines: $1 \leq i \leq n_0 = 2n - (l_1 + l_2)$ (where l_1 and $l_2 < l_1$ are the least in the sum positive integers obeying the inequality $n \leq l_2(l_1 + 1)$) and comprising for each $1 \leq i \leq n-1$ all those combinations of symbols F and B for which the total number of F is not equal to the total number of B .

(2) Various second-variety changes of a group of fixed power n result in the appearance of words represented by a column consisting of an even number of lines j : $2 \leq j \leq n-1$, if n is odd; $2 \leq j \leq n-2$, if n is even, the total number of F symbols always being equal to that of B symbols.

(3) Various third-variety changes of a group of fixed power n result in the appearance of words represented by a column consisting of an even number of lines k : $2 \leq k \leq n-2$, if n is even, and $2 \leq k \leq n-1$, if n is odd, — for the reference SCs; and $2 \leq k \leq 2(n-2)$ — for the non-reference SCs.

Proof

First let us consider statement (1). Let us prove the validity of the first part of this statement, i.e. prove that various first-variety changes result in the appearance of words represented by a column consisting of $1 \leq i \leq n_0 = 2n - (l_1 + l_2)$ lines, where l_1 and $l_2 < l_1$ are integers obeying the inequality so that $l_1 + l_2 \rightarrow \min$. To this end consider an allowed SC consisting of $l_1 = 1, 2, \dots, n$ SD rings, the first ring being of order $K_1^{(1)} \geq 1$, the second ring $-K_2^{(1)} \geq 1$, ..., the l_1 -th ring $-K_{l_1}^{(1)} \geq 1$, $\sum_{i=1}^{l_1} K_i^{(1)} = n$. Assume that a group of neurons can change from this SC to another allowed SC comprising $l_2 = 1, 2, \dots, n$ SD rings of order $K_1^{(2)} \geq 1$, $K_2^{(2)} \geq 1$, ..., $K_{l_2}^{(2)} \geq 1$, $\sum_{i=1}^{l_2} K_i^{(2)} = n$. This change corresponds to the formation of l_2 indistinguishable boxes of composition $K_1^{(2)}, K_2^{(2)}, \dots, K_{l_2}^{(2)}$ due to transfers of identical items accommodated in l_1 indistinguishable boxes of composition $K_1^{(1)}, K_2^{(1)}, \dots, K_{l_1}^{(1)}$. In so doing it must be stressed that in the course of the whole operation each item can be moved only once. Assume for definiteness that $l_1 > l_2$. Hence, $(l_1 - l_2)$ boxes shall be liquidated due to transfers of their items. But their liquidation, as may be inferred from Statement 2, will result in the appearance of $l_1 - l_2$ F symbols in the column-word formed during the change from one SC to another. Assume further that $t_1 \geq 0$ items are taken from the first box (so as not to leave it empty), $t_2 \geq 0$ — from the second box, ..., $t_{l_1} \geq 0$ — from the l_1 -th box. Then, as a result of the whole operation, the total number of lines in the column-word thereby formed will be $l_1 - l_2 + 2 \sum_{i=1}^{l_1} t_i$. Assume that all t_i items taken from a certain i -th box, $i = 1, 2, \dots, l_1$, are transferred to the different boxes. It is precisely this fact that furnishes maximum possible number of lines in the column-word at fixed l_1 and l_2 . Let us find the condition under which our assumption is valid. For the first box this condition will take the form $K_1^{(1)} - 1 \leq l_2$, for the second box — $K_2^{(1)} - 1 \leq l_2$, ..., for the l_1 -th box — $K_{l_1}^{(1)} - 1 \leq l_2$. Add up separately left-hand and right-hand sides of the inequality. This yields $n \leq l_2(l_1 + 1)$. But the fulfilment of this condition will give the following maximum possible number of lines in the column-word (when $t_i = K_i^{(1)} - 1$): $l_1 - l_2 + 2 \sum_{i=1}^{l_1} (K_i^{(1)} - 1) = 2n - (l_1 + l_2)$. From this expression it is easily seen that its maximum will be brought by l_1 and $l_2 < l_1$, being the least in the sum and obeying the inequality $n \leq l_2(l_1 + 1)$, which was to be proved.

Proceed now to the second part of statement (1). It will be proved, if for each given $1 \leq i \leq n-1$ we manage to find the allowed SCs the changes between which result in the appearance of words containing all combinations of F and B symbols, with the

total number of F different from that of B . First consider the case when a word consists only of F or only of B symbols. The appearance of such word represented by a column of F symbols having i lines, $i = 1, 2, \dots, n-1$, is effected by the change from the reference SC^(n) to the reference SC^($n-1$), SC^($n-2$), ..., SC⁽²⁾, SC⁽¹⁾. The reverse changes produce the word represented by a column of B symbols. The first-variety changes do not give any other words consisting of the same symbols. Consider now the case when the column representing a word comprises both F and B symbols. Let the column have an odd number of lines $1 \leq i \leq n-1$. It is, therefore, necessary to indicate SCs, the changes between which produce the column containing one B symbol and $(i-1)$ F symbols; two B symbols and $(i-2)$ F symbols; ...; and, finally, $(i-1)/2$ B symbols and $(i+1)/2$ F symbols. Apparently, the reverse changes produce the words which would form if in the columns corresponding to the direct changes all F symbols are substituted by B symbols and vice versa. If, on the contrary, the column has an even number of lines $2 \leq i < n-1$, it is necessary to indicate SCs, the changes between which produce the column containing one B symbol and $(i-1)$ F symbols; two B symbols and $(i-2)$ F symbols; ...; and, finally, $(i-2)/2$ B symbols and $(i-2)/2$ F symbols. It should be noted that, according to Statement 2, the column cannot contain equal number of F and B symbols.

Indicate two SCs, the change between which produces the column $[BFF \dots FF]^T$, where the sign T stands for the transposition. The initial SC is $l_1 = n-1$ of composition $1, 1, \dots, 1, 2$. The final SC is $l_2 = 2$ of composition $1, n-1$. At that, in the initial SC the SD ring of order 2 breaks inevitably. Consider two SCs, the change between which produces the column $[BBFF \dots FF]^T$. The initial SC is $l_1 = n-2$ of composition $1, 1, \dots, 1, 2, 2$. The final SC is $l_2 = 3$ of composition $1, 1, n-2$. In the general case for a column consisting of k B symbols

$\left(\text{where } 1 \leq k \leq \frac{(i-1)}{2}, \text{ if } i \text{ is odd and } 1 \leq k \leq \frac{(i-2)}{2}, \text{ if } i \text{ is even} \right)$ and $(i-K)$ F symbols we can, proceeding in a

similar way, indicate the following two SCs. The initial one: $l_1 = n-K$ of composition $1, 1, \dots, 1, 2, 2, \dots, 2, 2$ ($(n-2k)$ units and k twos). The final one: $l_2 = k+1$ of composition $1, 1, \dots, 1, 1, n-(k-1)$ (k units). During this change in the initial SC all SD rings of order 2 must break. Thus, statement (1) is fully proved.

Consider now statement 4(2). According to Statement 2, which was proved earlier, the second-variety changes correspond to the appearance of a word represented by the column having an even number of lines, the total number of F symbols being equal to that of B symbols. Consequently, to make Statement 2 fully proved, it remains only to prove that different second-variety changes produce the words represented by a column having an even number of lines j : $2 \leq j \leq n-1$, if n is odd; $2 \leq j \leq n-2$, if n is even. The second-variety changes are known to fit the condition $l_1 = l_2$ and $l_1 \neq 1, n, n-1$. This follows from Statement 3. Therefore, the maximum number of SD rings in the SC capable of the second-variety change equals $n-2$. This SC is the reference SC^($n-2$). The only SC to which a group of neurons can change from this SC (meaning the second-variety change) will be $l_2 = n-2$ of composition $1, 1, \dots, 1, 2, 2$. It will be

recalled that $k_1^{(1)} \leq k_2^{(1)} \leq \dots \leq k_{l_1}^{(1)}$ and $k_1^{(2)} \leq k_2^{(2)} \leq \dots \leq k_{l_2}^{(2)}$. But this change will produce the column $[BF]^T$, if the SD ring of order 3 breaks into three SD rings of order 1, 1, 1. The reference $SC^{(n-3)}$ has the number of SD rings equal to $l_1 = n - 3$ of composition 1, 1, ..., 1, 4. This SC can change to the SC having $l_2 = n - 3$ SD rings of order 1, 1, ..., 1, 2, 3. The change results in the appearance of the following columns: $[BF]^T$ when the SD ring of order 4 breaks into two rings, $[BBFF]^T$ when it breaks into three rings, $[BBBFFF]^T$ when it breaks into four rings, etc. For the reference $SC^{(n-k)}$ we have $l_1 = n - k$ of composition 1, 1, ..., 1, $k + 1$. In case of the second-variety change this SC produces the following columns: $[BF]^T$, $[BBFF]^T$, $[BBBFFF]^T$, ..., $[BB \dots BBFF \dots FF]^T$, the last of the columns containing the number of B symbols equal to that of F symbols and equal to k . The maximum possible value of k is defined proceeding from the following condition: $n - (k + 1) = k$, if n is odd and $n - (k + 1) = k + 1$, if n is even. This condition can be interpreted as follows: if n is odd, the number of SD rings of order 1 belonging to the initial reference $SC^{(n-k)}$ shall be equal to the order of sole SD ring, whose order exceeds unity, minus 1. Otherwise, the second-variety change cannot take place. If n is even, the number of SD rings of order 1 shall be exactly the same as the order of the sole SD ring whose order exceeds unity. But in so far as the total number of lines in the column is doubled (see Statement 2), we have the number of lines as required by statement (2). Thus, statement (2) is fully proved.

Proceed now to statement 4(3). Consider first the third-variety change for reference SCs. It is clear that reference stochastic conditions $SC^{(n)}$ and $SC^{(l)}$ which are not capable of this change shall be withdrawn from the analysis. As is easily seen, for the $SC^{(n-1)}$ the third-variety change results in the appearance of the column $[BF]^T$, for the $SC^{(n-2)}$ - $[BBFF]^T$, etc. However, there exists the lower bound (not equal to unity) of the number of SD rings belonging to a reference SC capable of the third-variety change. Let us find this bound. Consider a reference stochastic condition $SC^{(n-k)}$ for a neuron group of power n . In n is odd, the minimum $n - k$, still capable of the third-variety change, shall obey the equality $n - k - 1 = n - (n - k - 1) - 1$. Here $n - k - 1$ is the total number of SD rings of order 1, $n - (n - k - 1) = k + 1$ is the order of the sole SD ring belonging to the SC whose order exceeds unity. Then we have $k = (n - 1)/2$. Therefore, the number of B symbols

will be equal to $n - \left(\frac{n-1}{2}\right) - 1 = \frac{n+1}{2} - 1$. Consequently, the total number of lines

in the column produced by the third-variety change will be $2 \left(\frac{n+1}{2} - 1\right) =$

$n - 1$. This is the maximum number of lines, which the column-word can have at the third-variety change of a neuron group containing an odd number of elements n . Similar reasoning for the case of even n gives the following relationships:

$n - k - 1 = n - (n - k - 1)$. From this it is inferred that $k = \frac{n-2}{2}$. The number of

B symbols is $n - (n - k - 1) - 1 = \frac{n-2}{2}$. The total number of lines in the column

is $2 \left(\frac{n-2}{2}\right) = n - 2$.

Consider now non-reference SCs which contain 1 SD rings of order $k_1 \leq k_2 \leq \dots \leq k_l$, $\sum_{i=1}^l k_i = n$, at least two of these rings having the order exceeding unity; $1 \geq k_1$, otherwise the third-variety change is not feasible. Apparently, the corresponding column-word produced by the change will contain the more lines, the more SD rings of the given SC will break. This suggests that the maximum number of B symbols will appear when all rings whose order exceeds unity will break, down to the rings of the first order. This is the right time to recall that the third-variety changes retain the number of the initial and final SCs, i.e. produce the column consisting of an even number of lines (see Statement 2), such that the total number of B symbols equals that of F symbols. Hence, the ring of order k_1 will "yield" $k_1 - 1$ B symbols; the ring of order k_2 will "yield" $(k_2 - 1)$ B symbols, ..., the ring of order k_l will "yield" $(k_l - 1)$ B symbols. The total number of B symbols (the maximum one for the initial SC of the given composition) appearing at the third-variety changes will be $\sum_{i=1}^l (k_i - 1)$. Then the total number of lines in the word will be

$$2 \sum_{i=1}^l (k_i - 1) = 2 \left(\sum_{i=1}^l k_i - l \right) = 2(n - l), \quad l \neq 1, n,$$

(because $l = 1$ is nothing but the reference $SC^{(l)}$, whereas $l = n$ is the reference $SC^{(n)}$, and these SCs, as was already mentioned, are not capable of the third-variety change). Should we try to vary l , we will have $\max 2(n - l) = 2(n - 2)$ and $\min 2(n - l) = 2(n - (n - 1)) = 2$. In other words, the first result is the maximum of the maximum possible breaks of SD rings, whereas the second result is the minimum of the same. Thus, statement (3) is fully proved. Thereby Statement 4 is also proved.

Note

Each fixed n (the group power) corresponds to its own minimum value of $l \geq k_l$, where k_l is the maximum order of the SD ring belonging to an allowed for this group SC which have the maximum number of lines in the column-word at the third-variety change. That is why the value $2(n - 2)$ represents only the upper bound of the number of lines in the column-word, not necessarily attainable for all values of n .

Taking this into account, let us find the upper bound of the number of words for a neuron group of power $n \geq 3$ (this limitation may be attributed to the fact that the group of power $n = 2$ features neither rotation nor degeneration). According to Statement 4, the 1st, 2nd and 3rd-variety changes (of reference SCs) result in the situation when any number of lines, within the range from 1 to n_0 inclusive, can be juxtaposed with a column-word. Estimate the value of n_0 . To do this solve the following problem: find the minimum of the value

$$F = l_1 + l_2$$

with the following limitations:

$$(8) \quad l_2 < l_1, \quad n \geq l_1, \quad l_2 \geq 1$$

$$(9) \quad n \leq l_1(l_2 + 1)$$

l_1 and l_2 being integers.

The solution:

$$1. \text{ Let } l_2 \geq [\sqrt{n}] + 1^*, \text{ then } \frac{n}{l_2 + 1} \leq \frac{n}{[\sqrt{n}] + 2} < [\sqrt{n}] + 1, \text{ since } n < ([\sqrt{n}] + 1)^2.$$

Therefore, l_1 is taken from limitation (8), i.e. we have the problem

$$F = l_1 + l_2 \rightarrow \min$$

$$l_2 < l_1$$

$$[\sqrt{n}] + 1 \leq l_2 \leq n$$

Its solution takes the form

$$l_2 = [\sqrt{n}] + 1,$$

$$l_1 = l_2 + 1 = [\sqrt{n}] + 2,$$

$$F = 2[\sqrt{n}] + 3.$$

2. Let now $l_2 \leq [\sqrt{n}]$, then

$$\frac{n}{l_2 + 1} \geq \frac{n}{[\sqrt{n}] + 1} > [\sqrt{n}] - 1,$$

since

$$([\sqrt{n}] + 1)([\sqrt{n}] - 1) = [\sqrt{n}]^2 - 1 < n.$$

Therefore, l_2 is taken from limitation (9), i.e. we have the problem

$$F = l_1 + l_2 \rightarrow \min$$

$$l_1 \geq \frac{n}{l_2 + 1}$$

$$1 \leq l_2 \leq [\sqrt{n}]$$

It is solvable at the minimum l_1 , that is at

$$l_1 = \left[\frac{n}{l_2 + 1} \right] + 1.$$

This consideration enables the following problem to be formulated:

$$F = \left[\frac{n}{l_2 + 1} \right] + 1 + l_2 \rightarrow \min$$

$$1 \leq l_2 \leq [\sqrt{n}]$$

We now find its solution. Denote

$$\Phi(l_2) = \frac{n}{l_2 + 1} + 1 + l_2.$$

Then, obviously,

$$[\Phi(l_2)] = F(l_2),$$

$$\Phi'(l_2) = \frac{-n}{(l_2 + 1)^2} + 1.$$

Make the derivative equal to zero and determine l_2 from the expression obtained. It yields $l_2 = \sqrt{n} - 1$. Hence $\Phi(l_2)$ decreases at all $l_2 \in (0, \sqrt{n} - 1)$ and, specifically, at $l_2 \in (0, [\sqrt{n}] - 1)$. From this it follows that

$$[\Phi(1)] \geq [\Phi(2)] \geq \dots \geq [\Phi([\sqrt{n}] - 1)].$$

Also, if \sqrt{n} is an integer, the function $\Phi(l_2)$ will decrease at $l_2 \in (0, [\sqrt{n}])$. Thus, the solution is obtained:

$$\text{at } l_2 = [\sqrt{n}], \text{ if } \sqrt{n} \text{ is an integer,}$$

$$\text{at } l_2 = [\sqrt{n}] - 1, \text{ if } \sqrt{n} \text{ is not an integer.}$$

Proceeding further, one can state that the following estimates are valid:

$$F([\sqrt{n}]) = \left[\frac{n}{[\sqrt{n}] + 1} \right] + 1 + [\sqrt{n}] \leq \frac{n}{[\sqrt{n}] + 1} + 2 + [\sqrt{n}]$$

$$\leq [\sqrt{n}] + 1 + 2 + [\sqrt{n}] = 2[\sqrt{n}] + 3.$$

$$F([\sqrt{n}] - 1) = \left[\frac{n}{\sqrt{n}} \right] + [\sqrt{n}] \leq \frac{n}{[\sqrt{n}]} + [\sqrt{n}] \leq [\sqrt{n}] + 3 + [\sqrt{n}] = 2[\sqrt{n}] + 3.$$

Therefore, as it follows from clauses 1 and 2, the value of F does not exceed $2[\sqrt{n}] + 3$. Consequently, $2n - (l_1 + l_2) = n_0 \geq 2n - 2[\sqrt{n}] - 3$.

As for the third-variety changes of non-reference SCs, they produce the column consisting of an even (and only even) number of lines, from 2 to $2(n-2)$ inclusive.

The "additional" number of words (appearing due to the third-variety changes of non-reference SCs), which can be acquired by a neuron group of power n is easy to find based on the following considerations. Let n_o be even. Then the "additional" (in the foregoing sense) words will consist of the following even number of lines: $n_o+2, n_o+4, \dots, 2(n-2)-2, 2(n-2)$. Their total number is equal to $\frac{2n-n_o-2}{2}$. Let n_o be odd. Then the "additional" words will comprise the following number of lines: $n_o+1, n_o+3, \dots, 2(n-2)-2, 2(n-2)$. Their total number is equal to $\frac{2n-n_o-3}{2}$.

Hence, the changes of all three varieties result in the number of words (the vocabulary) not exceeding:

$$W_n = \sum_{i=1}^{n_o} \frac{(1+i)!}{i!} + \frac{2n-n_o-3}{2}, \quad \text{if } n_o \text{ is odd}$$

$$\sum_{i=1}^{n_o} \frac{(1+i)!}{i!} + \frac{2n-n_o-2}{2}, \quad \text{if } n_o \text{ is even.}$$

From this expression it is clear that the overwhelming majority of the words appear due to the 1st, 2nd and 3rd (of reference SCs) variety changes. Table 1 gives the idea of the total number of words W_n and of the number of words appearing due to the 3rd-variety changes of non-reference SCs. Table 2 gives the idea of the structure of words appearing at the 1st, 2nd and 3rd variety changes of a neuron group of power $n=6$.

Assume finally that for the analysis of functioning of a neuron group m "sliding" intervals are used (these overlapping intervals are supposed to fully cover the fragment of the neurograms under consideration). Then the neuron group of power n possesses potentially the following volume of different "phases" $V(n, m)$:

$$V(n, m) = (W_n + 1)^{m-1}$$

Table 1

Total number of words W_n and the number of words appearing due to the 1st, 2nd and 3rd- (of reference SCs) variety changes depending on the neuron group power $n=6$.

n	3	4	5	6	7	8	9	10
W_n	5	10	15	22	29	38	47	58
$\sum_{i=1}^{n_o} \tilde{C}_2^i$	5	9	14	20	27	35	44	54

Here \tilde{C}_2^i is the number of combinations of 2 taken i at a time with repetitions.

Table 2

The structure of words appearing at the 1st, 2nd and 3rd-variety changes for a neuron group of power $n=6$ (only direct changes are shown; the reverse changes are described with the words obtained from the direct changes by substituting B for F and F for B).

Sl No.	Structure of change	Variety of change	Number of SD rings		Composition of initial SC	Composition of final SC	Word
			initial SC	final SC			
1	2	3	4	5	6	7	8
1		I	6	5	1,1,1, 1,1,1	1,1,1, 1,2	F
2		I	6	4	1,1,1, 1,1,1	1,1, 2,2	F F
3		II	4	4	1,1,1,3	1,1,2,2	F B
4		I	4	5	1,1,2,2	1,1,1, 1,2	F B B
5		I	6	3	1,1,1, 1,1,1	1,1,4	F F F
6		I	3	5	2,2,2	1,1,1, 1,2	P B B B
7		II	4	4	1,1,2,2	1,1,1,3	F F B B
8		I	6	2	1,1,1, 1,1,1	1,5	F F F F
9		I	6	4	1,1,1, 1,1,1	6	F F F F F
10		I	2	5	1,5	1,1,1, 1,2	F B B B B

Table 2 (Continuation)

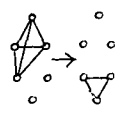
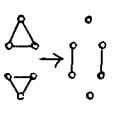
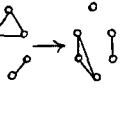
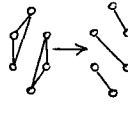
S1 No.	Structure of change	Variety of change	Number of SD rings		Composition of initial SC	Composition of final SC	Word
			initial SC	final SC			
1	2	3	4	5	6	7	8
11		I	3	4	1,1,4	1,1,1,3	F F B B B B
12		none	—	—	—	—	F F F F F F
13		none	—	—	—	—	F B B B B B
14		I	2	4	3,3	1,1,2,2	F F B B B B
15		III	3	3	1,2,3	1,2,3	F F F B B B
16		none	—	—	—	—	F F F F F F

Table 2 (Continuation)

S1 No.	Structure of change	Variety of change	Number of SD rings		Composition of initial SC	Composition of final SC	Word
			initial SC	final SC			
1	2	3	4	5	6	7	8
17		none	—	—	—	—	F B B B B B
18		none	—	—	—	—	F F B B B B
19		I	2	3	3,3	2,2,2	F F F B B B
20		none	—	—	—	—	F F F F B B

The addition of 1 to W_n is made assuming that there exists an "empty" word. The "empty" word appears when the adjacent "sliding" intervals (see Kovbasa *et al.*, 1984) identified one and the same SC, i.e. when neither of variety changes took place.

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