What to do with a given time series? — Practical Examples

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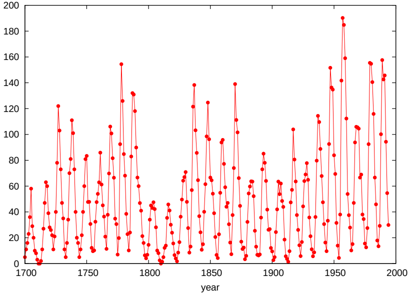
What to do with a given time series?

1. Simply look at the time series - at the graph $x(t)$ and at the delay plot.
2. Perform simple analysis:
   - Histogram - Gaussian or not?
   - Spectrum - typical frequencies?
   - Delay plot - signatures of nonlinearity?

The graph of the time series

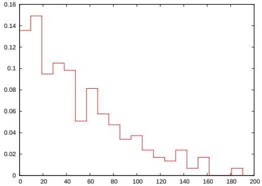
Example: Sunspots data set from Yao and Tong (1994)



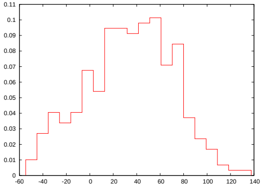
- Typical oscillations? Other typical patterns?
- How noisy are the data?
- Artefacts, outliers?
- Missing data?

Histogram of the data - Gaussian or not?

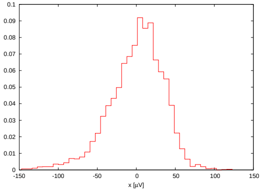
Sunspots



AR(2)



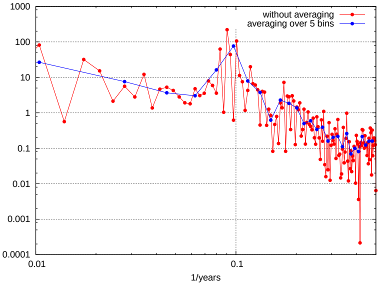
EEG

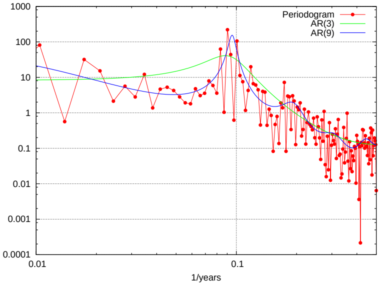


Test the distribution for Gaussianity - for instance with the Kolmogorov Smirnov test:

- Sunspots: Rejected
- AR(9): Not rejected
- EEG: Result depends on the number of data points used - rejection for large numbers

Spectral analysis

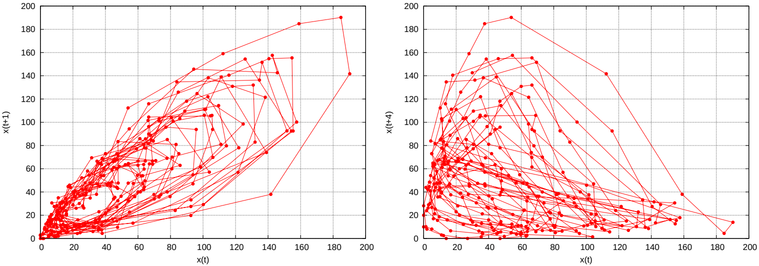




- *spectrum*: tradeoff between frequence resolution and fluctuations
- Typical period $\gtrsim$ 10 years
- *mem_spec* also useful for testing AR-model - AR(9) better than AR (2)

Delay plot

- Get a feeling for good delays from the mutual information or autocorrelation function.
- Try different delays!



- Nonlinear signatures: Look for "holes" in the attractor.

Linear vs. non-linear models

Possible criteria:

- Nonlinear signatures in the graph or delay plot
  Sunspots: Data not Gaussian ditributed, always > 0, at least non-linear measurement function
- Properties incompatible with linear models
  Sunspots: "hole" in the delay plot
- Number of points — non-linear methods need more
  Sunspots: Not enough points for most nonlinear methods
- local linear model performs better than global linear model
  Sunspots: see next slide
- Surrogate data test:
  Sunspots: Using *predict* as test statistic: Only for $d = 1$ $m = 3$ weak rejection of the null hypothesis

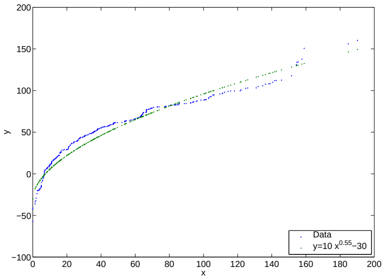
$\Rightarrow$ Some, but no clear evidence for non-linearity.

Linear modeling

- No convincing rejection of the null hypothesis of a linear process observed via a nonlinear measurement function
- Nonlinear measurement function essential because $x_n > 0 \quad \forall n$.
- Direct fit of a linear model to the data will give us not a good model
- First we have to transform the data to a Gaussian distribution

Transformed data

- Replacing data by Gaussian distributed data with the same rank order



- *lfo-ar*: Only for the original data the local linear model performs better than global linear model
- Optimal linear model according to AIC — original data $p = 9$, transformed data $p = 12$.
- Is the linear model for the transformed data better than on the orginal data? No!

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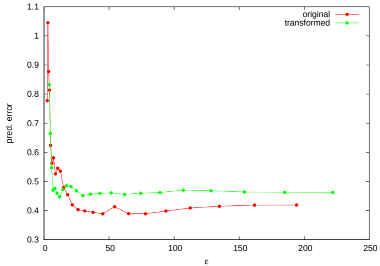

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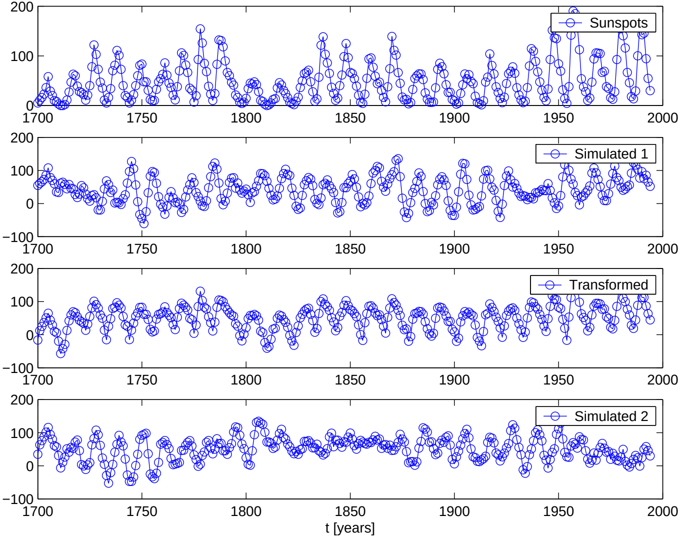
Prediction and Modeling

- AR-model: main period 10.8 (transformed data) vs. 10.5 years in the original data
- damping time between $\approx$ 40.8 and 39.3 years
- If the linear model is true, the sunspots will be only predictable within one cycle.

Prediction and Modeling

- The linear model is insufficient



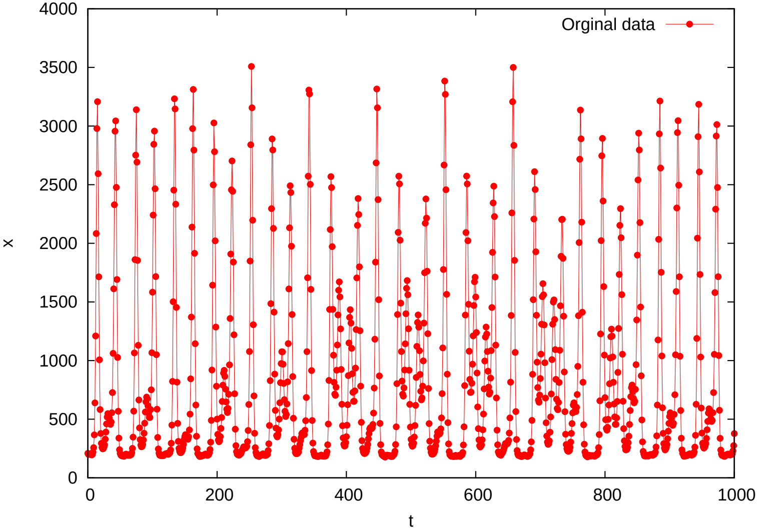
Prediction and Modeling

- AR-model: main period 10.8 (transformed data) vs. 10.5 years in the original data
- damping time between ≈ 40.8 and 39.3 years
- If the linear model is true, the sunspots will be only predictable within one cycle.
- The linear model is insufficient
- I could not find a convincing non-linear model, but there are proposals in the literature.

Data from a controlled physical laboratory experiment - the NMR laser data

- For a description of the data set and many results see Kantz/Schreiber
- NMR laser experiment at the Zurich University in the group of Prof. Brun
- Lasing particles are Al atoms in a ruby crystal
- To induce chaos the quality factor of the laser is periodically modulated

Part of the raw data



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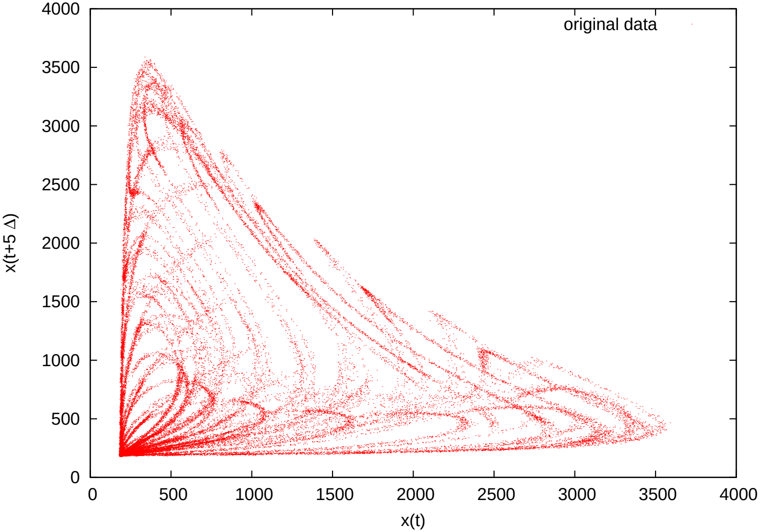
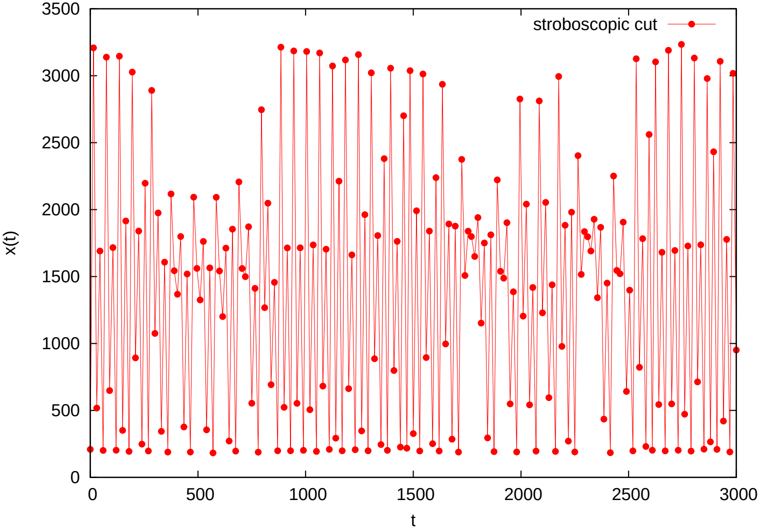


Stroboscopic cut with period of the driving



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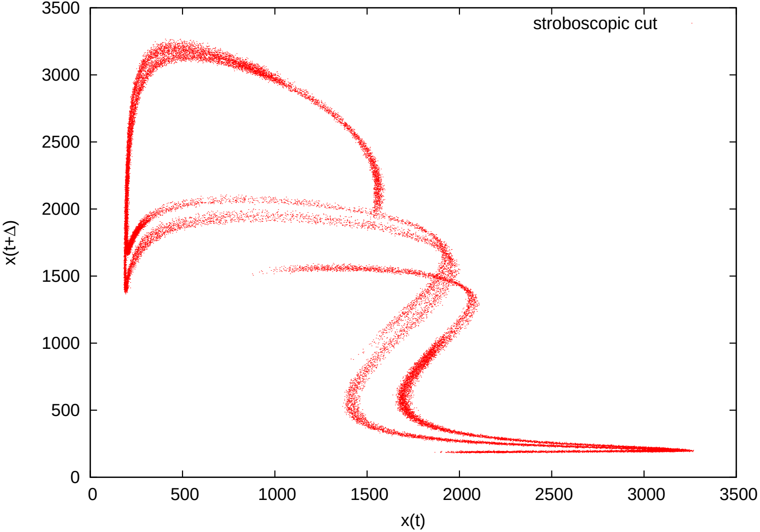
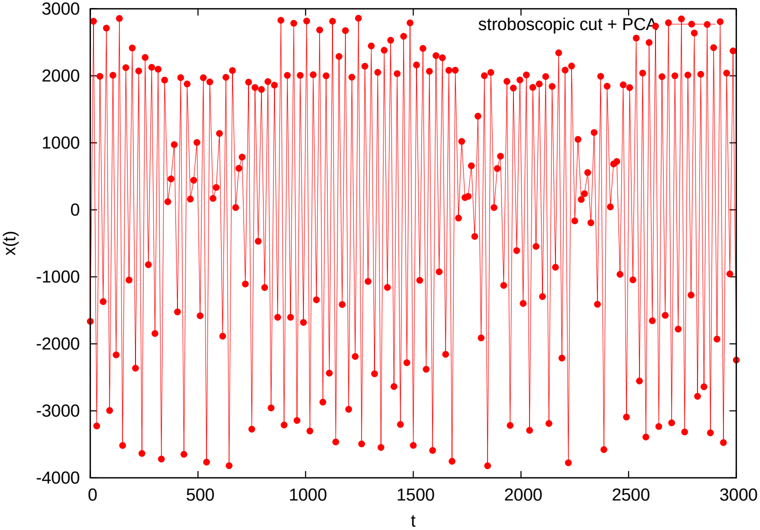


Projection on the largest PCA component



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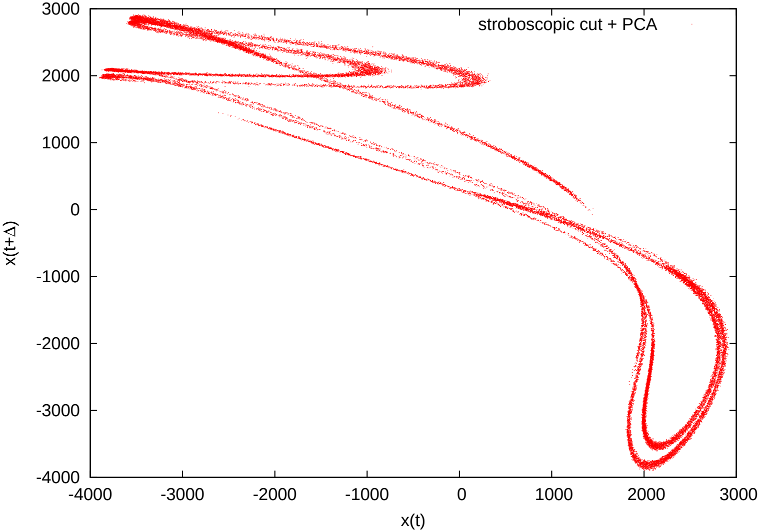
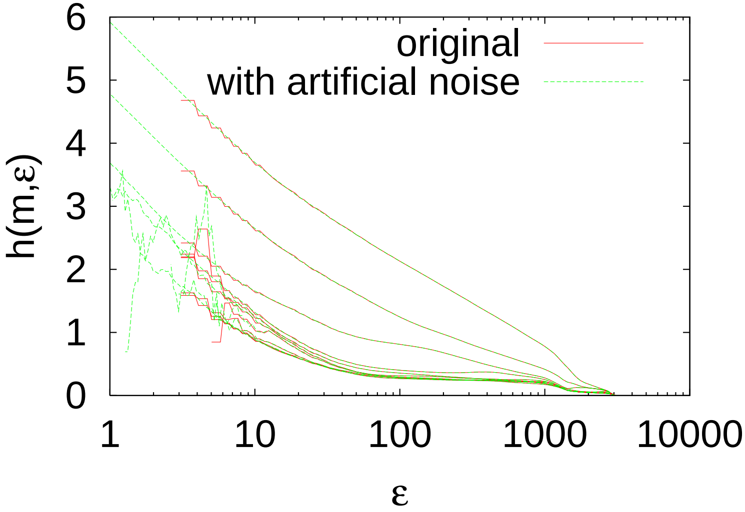
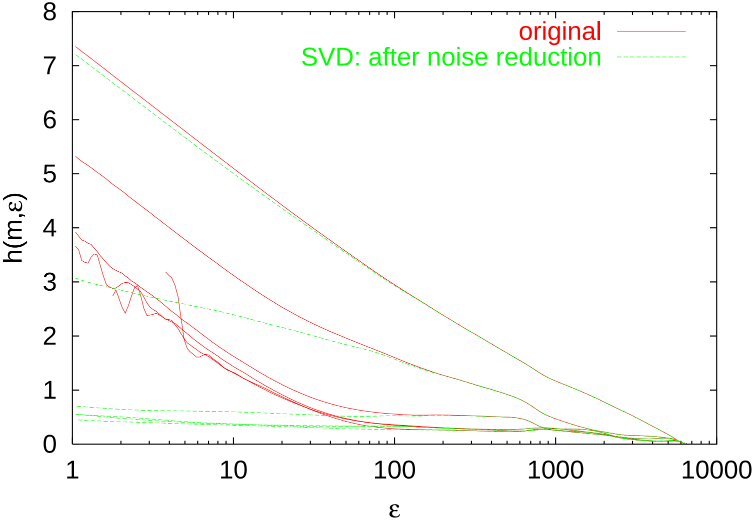


Entropies

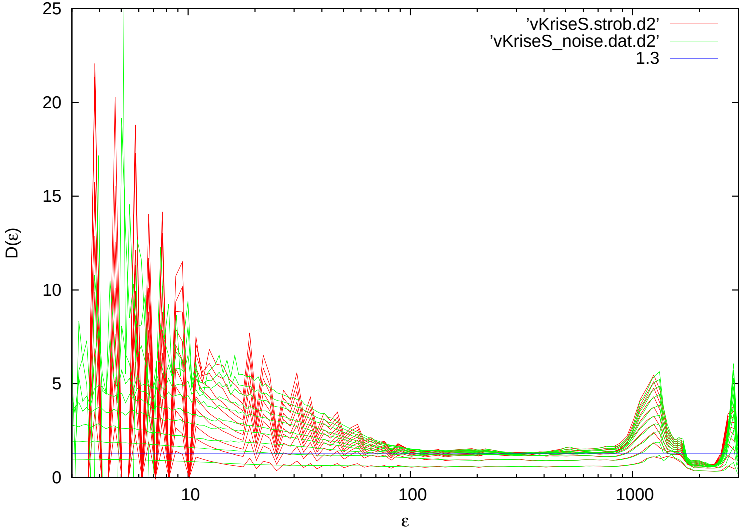

original
with artificial noise
h(m,ε)
ε
0
1
2
3
4
5
6
1
10
100
1000
10000

Entropies

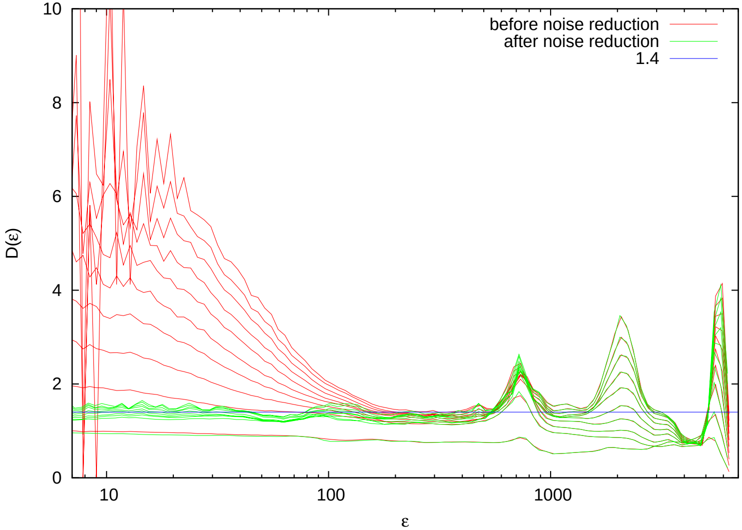


original
SVD: after noise reduction
h(m,ε)
ε
8
7
6
5
4
3
2
1
0
1
10
100
1000
10000

Correlation dimension

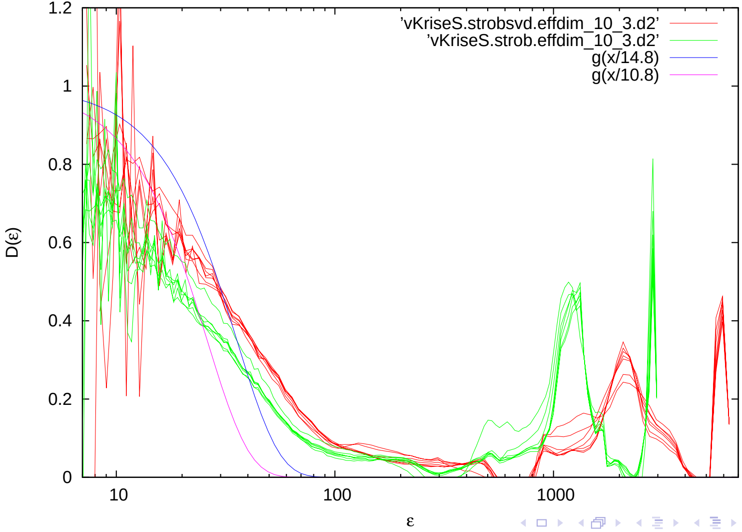


Correlation dimension



Noise level

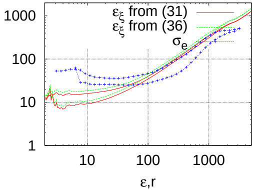
Assumption measurement noise:



Noise level

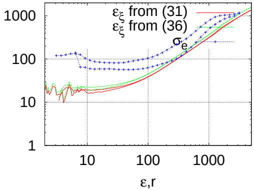
Assumption dynamical noise:
Noise level estimates and prediction error of a local linear predition with $m = 3, 4$.

Strob.



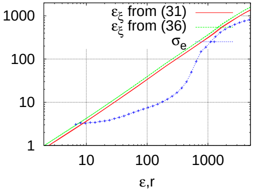
≥ 10

+PCA



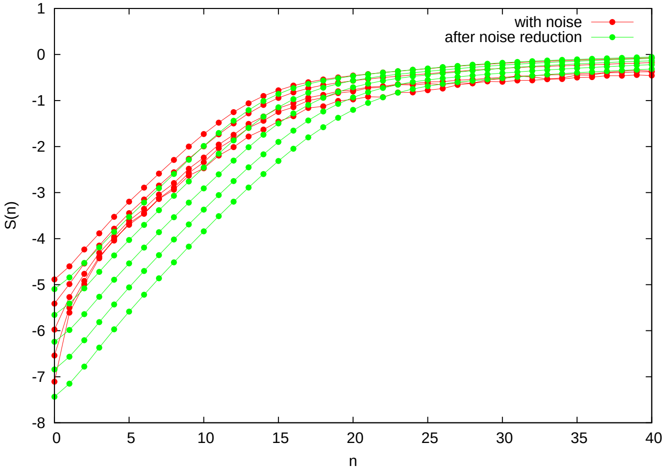
*approx*20

+Noise reduction



no noise level

Lyapunov exponents

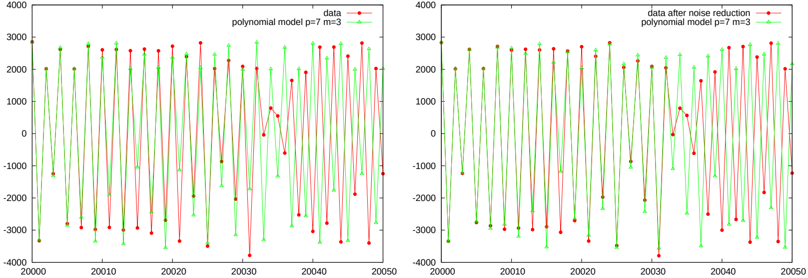


Noise reduction allows much better estimates! Largest Lyapunov exponent in the stroboscopic cut ≈ 0.3.

Consistency of the invariants

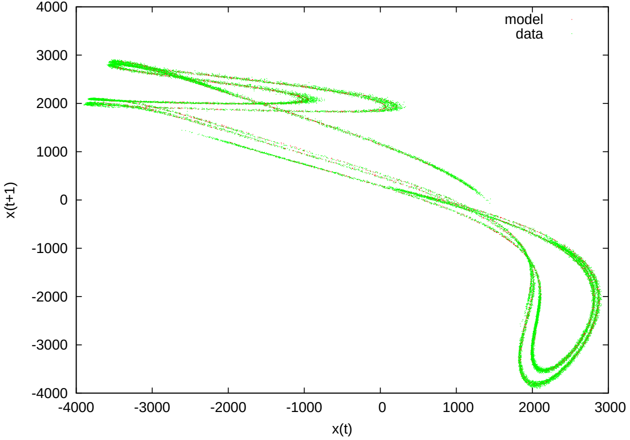
- Lyapunov expoents:
  - *lyap_k* $\lambda \approx 0.3$
  - *lyap_spec* $\lambda = 0.27 \ldots 0.3$
- Entropy:
  - *lyap_spec* $\lambda = 0.27 \ldots 0.3$
  - *d2*, i.e. correlation entropy: $h^{(2)}(m = 10) = 0.23 \ldots 0.4$
  - *boxcount* $h(m = 10) = 0.31 \ldots 0.32$
- Dimension: Should be smaller than 2!
  - *lyap_spec* $D_{KY} = 1.74 \ldots 2.8$ — not reliable!
  - *d2*, i.e. correlation dimension $D_2 = 1.3 \ldots 1.4$.
  - *boxcount* $D_1 = 1.34 \ldots 1.36$

Modeling



Good polynomial model already for noisy data.

Modeling

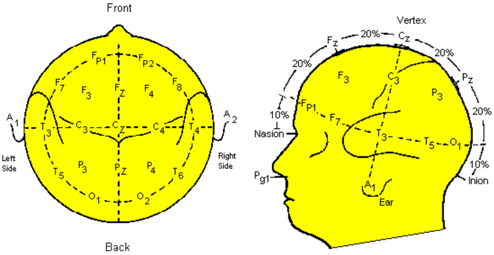


Good polynomial model already for noisy data.

Physiological data - human sleep EEG

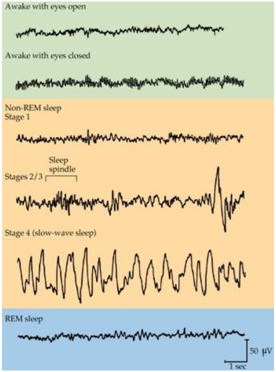
- Excitatory and inhibitory post-synaptic potentials (EPSP and IPSP) correspond to depolarization or hyperpolarization of the cell membrane, respectively.
- the EEG mainly originates from the summed dendritic extracellular changes in ion concentrations that result from chemically mediated EPSPs and IPSPs and last for about 10-250ms.
- accumulations of charge outside the dendrite cause electric currents that flow through the surrounding media (brain tissue, cerebrospinal fluid, skull and skin).
- electric currents change the electrical potentials on the scalp by Ohm's law due to the electrical resistance of the tissue.

EEG



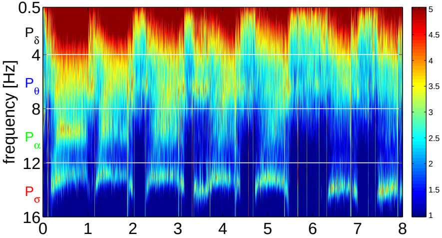
- Measuring potential differences on the surface of the scalp — voltage in the order of $10 - 200\mu$V.
- Advantage: High temporal resolution — same as MEG but much less expensive
- Disadvantage: Low spatial resolution

Sleep stages and sleep oscillations



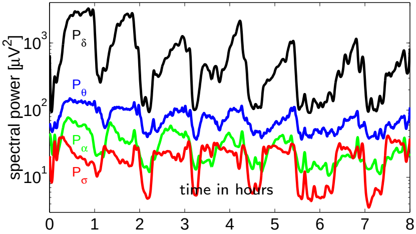
- Traditionally the EEG was recorded on paper with a velocity of 30 mm/s corresponding to the frequencies of the typical EEG Oscillations between 0.5 and 30 Hz.
- Sleep stages were defined with respect to certain typical oscillatory patterns in the sleep EEG, such as sleep spindles, K complexes and slow waves

Linear methods — Spectral analysis



Typical time scales:

- sleep oscillations 1-2 s
- sleep stages typically defined for 20 s segments
- non-REM/REM cycle ca. 80-90 min

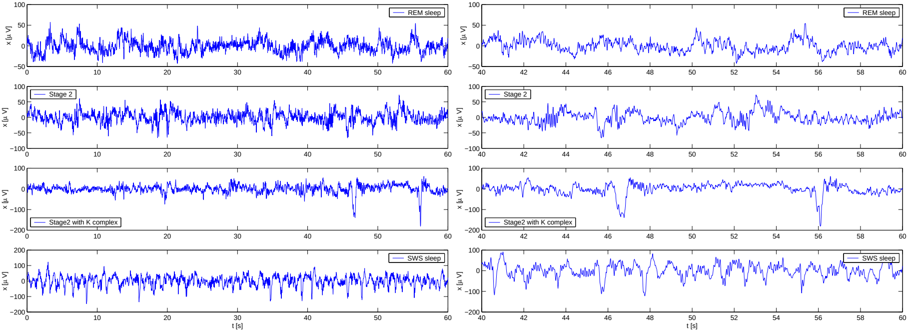


Spectral band power in different frequency bands:

δ ... 0 - 4 Hz
θ ... 4 - 8 Hz
α ... 8 - 12 Hz
σ ... 12- 16 Hz (sleep spindles)

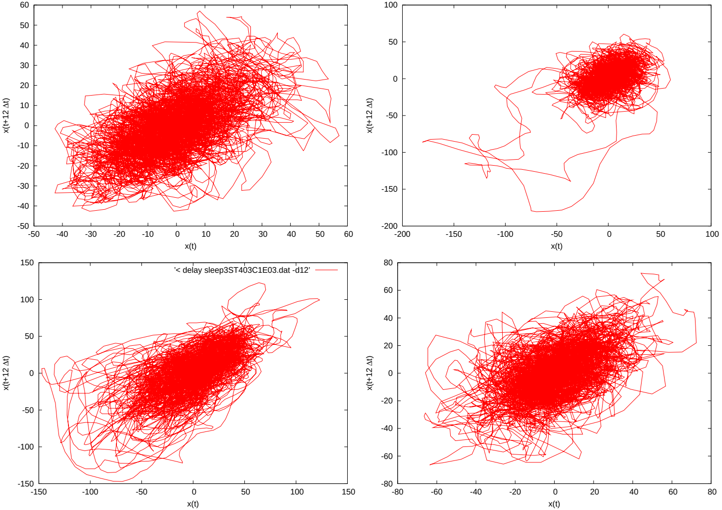
The data

Selected 60-s segments from different sleep stages

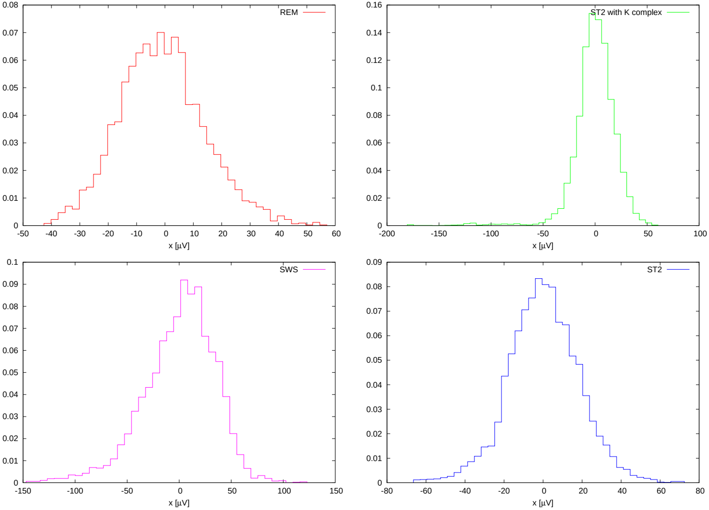


Time scales matter for visualization. Try different time scales!

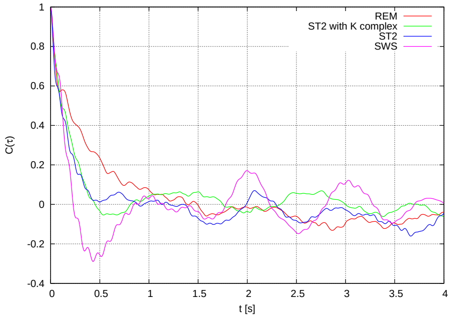
Delay plots

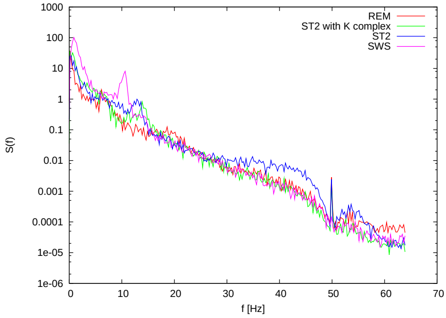


Histograms



Correlation function and power spectrum





What could be learned so far?

- Almost Gaussian distributed amplitudes, but slight assymmetry of their distribution
- No clear deterministic signatures found
- Stronger correlations in deeper sleep stages
- Oscillatory patterns with spectral signatures - alpha oscillations (≈ 10 Hz) in deep sleep, spindles oscillations ($\approx 12 - 14$ Hz) in stage 2 sleep
- Further questions: Linear, non-linear or non-stationary?

Exkurs: Deterministic chaos in the sleep EEG?

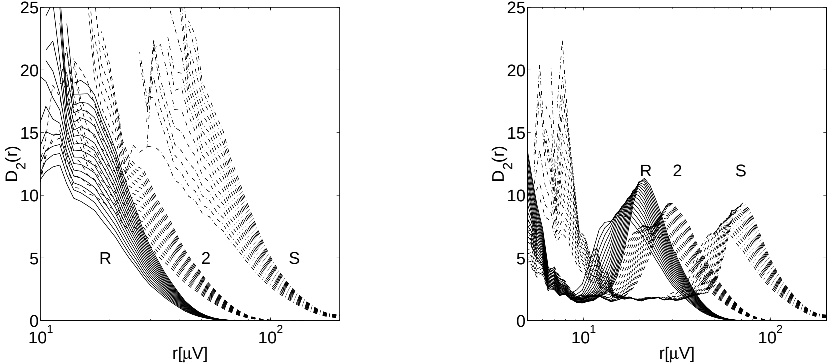
Published correlation dimensions of sleep EEG:

authors	Babloyantz (85)	Röschke(92)	Pradhan(95)	Rey(96)
nr. of subjects	3	12	5	9
awake			8.7 ± 0.14	7.59 ± 0.24
REM		6.17 ± 0.37	9.20 ± 0.16	7.24 ± 0.1
stage 1			7.26 ± 0.21	7.36 ± 0.14
stage 2	5.1	5.87 ± 0.37	6.88 ± 0.32	6.66 ± 0.07
stage 3		4.72 ± 0.31	5.36 ± 0.22	5.31 ± 0.11
stage 4	4.16	4.37 ± 0.27	4.45 ± 0.12	4.49 ± 0.1

- decreasing dimension with deeper sleep
- same order of magnitude within one sleep stage
- evidence for chaos?

Finite Dimensions due to temporal correlations

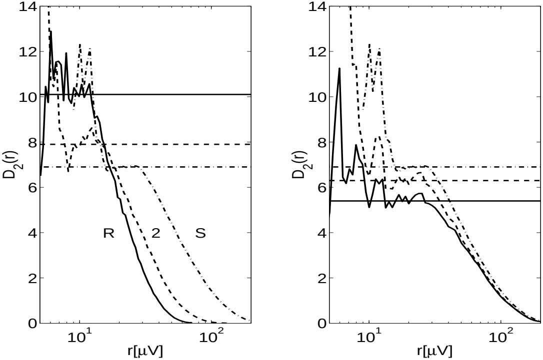
$$C(r) = \frac{2}{(N-W)\cdot(N-1-W)} \sum_{i=1}^{N} \sum_{j=i+1+W}^{N} \Theta(r - ||\vec{X}_i - \vec{X}_j||) \quad D_2(r) = \frac{d \log C(r)}{d \log r}$$



with Theiler correction $W = 50\Delta$, delay 20Δ

without Theiler correction, delay 20Δ

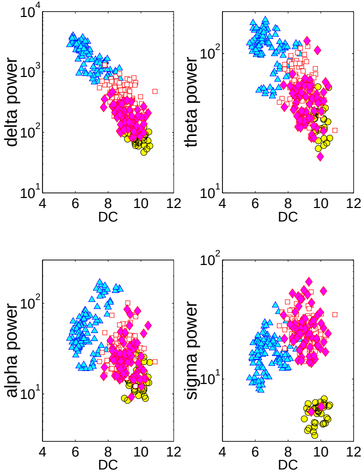
Finite Dimensions from temporal correlations II



Theiler correction $W = 1\Delta$, Delay 3Δ (left) and $3, 5, 10\Delta$, only slow wave sleep (right)

- spurious plateaus from temporal correlations (Theiler 1986, Osborne and Provenzale 1989)
- fractional Brownian motion $P(f) \propto f^{-\beta} \quad \Rightarrow \quad D = 2/(\beta - 1)$

Correlation dimension can be explained by delta power



Different colors denote different sleep stages:
yellow ... REM sleep
red, pink ... light sleep (stage 2)
red squares ... with K complexes
pink diamonds ... without K complexes
turquoise ... Slow wave sleep (SWS)

Evidence for nonlinearity in the sleep EEG?

Surrogate data tests:

	Segment length	Surrogates	Test statistic	Sleep stage
Fell et al. 1996	20.5-164-s	AAFT	D_2, L_{max}	
Pereda et al. 1998	16-s	AAFT	D_2	SWS
Shen et al. 2003	60-s	IAAFT	D_2	stage 2

AAFT: amplitude adjusted Fourier Surrogates (Theiler92)
IAAFT: iterated AAFT (Schreiber and Schmitz 2000)

Effects of the segments length

- Surrogates from AR-models with randomly shuffled residuals
- Test statistic: Correlation sum at $r = 0.5\sigma$.

Fraction of subsegments for which the null hypothesis is rejected:

T [s]	REMS	Stage 2A	Stage 2B	SWS
1	0.35%	0.52%	0.43%	0.22%
2	0.58%	2.79%	0.80%	0.41%
4	0.82%	10.08 %	2.78%	1.15%
10	3.8%	16.12%	13.37%	4.37%
30	8.77%	88.95%	49.48%	18.85%
60	19.3%	86.05%	67.71%	44.26%

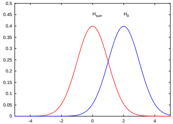
$\Rightarrow$ No nonlinearity for $T = 1$ s. More frequent rejection of the null hypothesis with increasing segment length T.
But: the rejection of the null hypothesis might be due to nonlinearity **or** nonstationarity.

Nonlinearity or Nonstationarity?

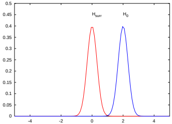
stationary nonlinearity

$\Rightarrow E(\langle H_{surr} \rangle - H_0)$

independent of T



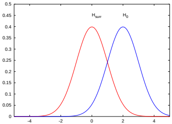
$\Rightarrow$



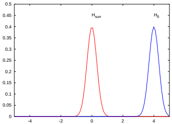
nonstationarity

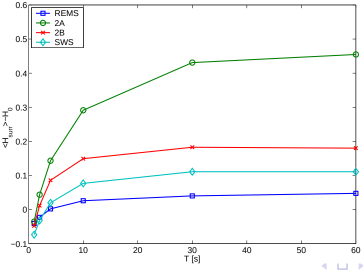
$\Rightarrow E(\langle H_{surr} \rangle - H_0)$

increases with T



$\Rightarrow$





Time-varying autoregressive (TVAR) model

- Surrogate data analysis: nonlinearity in human sleep EEG (single channel) due to **non–stationarity** $\Rightarrow$ on short segments ($\approx 1s$) **linear**
- **Autoregressive (AR) model** on overlapping 1-s segments

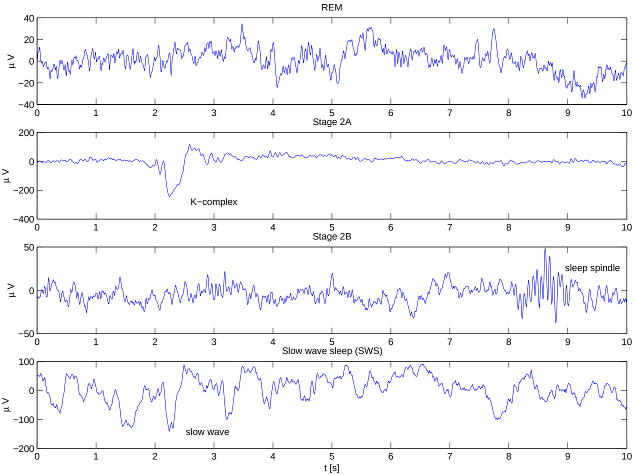
$$x_n = \sum_{k=1}^{p} a_k x_{n-k} + \xi_n \quad \text{in matrix form} \quad \boldsymbol{X}_{n+1} = A\boldsymbol{X}_n + \boldsymbol{\xi}_{n+1}$$

- Diagonalization of A $\rightarrow$ Eigenvalues $z_k = r_k \exp(-i\phi_k)$
- stochastically driven harmonic oscillators with **damping constants** (Δ ... sampling interval)

$$\gamma_k = \tau_k^{-1} = -\Delta^{-1} \ln \mathbf{r_k} \quad \text{and } \textbf{frequencies} \quad \mathbf{f_k} = \phi_k/(2\pi\Delta)$$

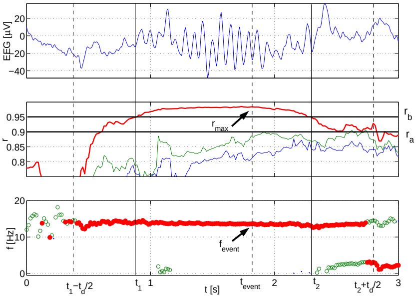
- time dependent frequencies $f_k(t)$ and damping constants $\gamma_k(t)$ on time scales > 1 s

Sleep Oscillations



- sleep stages are defined via oscillatory events (K-complexes, sleep spindes, slow waves)
- time scale of these events $\approx 1 - s$

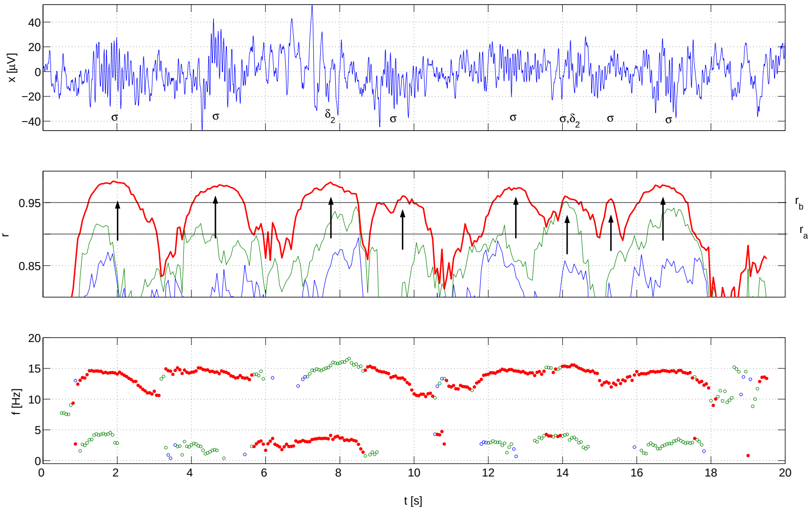
Detection of oscillatory events



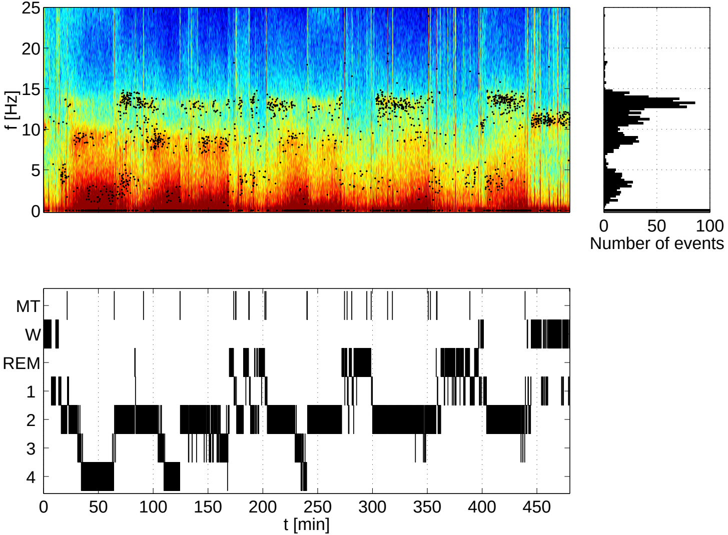
- AR(8)-model
- Detection thresholds:
  $\mathbf{r_k} > \mathbf{r_b} = 0.95.$
  $t_1 \leq t \leq t_2$ with
  $\mathbf{r(t)} > \mathbf{r_a} = 0.9$ and
  $\mathbf{r(t_1)} = \mathbf{r(t_2)} = \mathbf{r_b}$

- Corresponding damping times with sampling frequency 128 Hz:
  $\boldsymbol{\tau_b} = 0.152$ s, $\boldsymbol{\tau_a} = 0.074$s
- Event time $\mathbf{t_{event}}$:
  $r(t_{event}) = r_{max}$
- Event damping:
  $\gamma = \tau^{-1} = -\Delta^{-1} \ln \mathbf{r_{max}}$
- Event frequency:
  $\mathbf{f_{event}} = f(t_{event})$
- Event duration:
  $\mathbf{T_{event}} = t_2 - t_1 + 1$s

Example from sleep stage 2 with sleep spindles

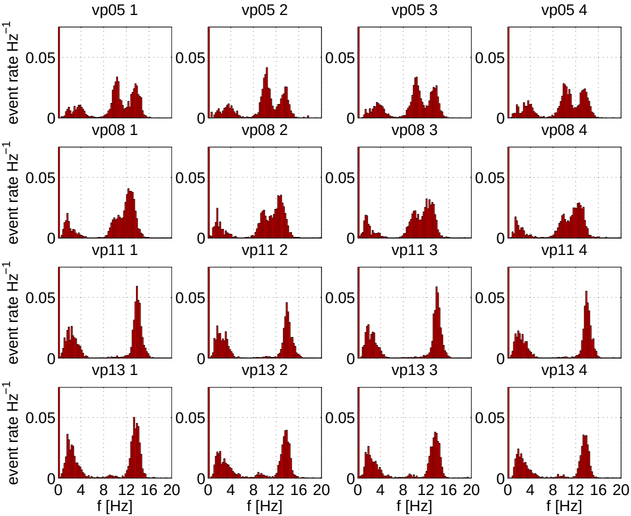


Events during one night



Individually typical frequency distribution of the events

4 nights of 8 h duration each for each subject



Applications of this analysis

- Sleep stage dependencies of event densities and frequencies
- Role of the events in sleep regulation: change after sleep deprivation
- Changes in pathological conditions — epilepsy, depression
- Hypothesis: Oscillatory events correspond to resonances of the underlying neural networks

Summary

1. What kind of data do I have? What do I know about the data?
   - How much data?
   - Typical time scales? Stationarity?
   - Noise? What kind of noise?
2. Simple analysis: graph of the time series, delay plot, histogram, correlation function and spectrum
   - Amplitudes Gaussian distributed?
   - Nonlinear signatures?
3. Linear, non-linear or non-stationary? If no clear answer - surrogate data test.
   - If nonlinear: noise reduction if nessecary, estimating invariants (entropy, dimension, Lyapunov exponents), modeling $\Rightarrow$ consistency check
   - If linear: linear models, spectral analysis
   - Non-stationary: time-frequency methods, such as spectrogram, wavelet analysis, linear models with time dependent parameters, ...